Analyzing Environmental Policies with IGEM, an Intertemporal General Equilibrium Model of U.S. Growth and the Environment Part 2

Appendix A. Equations of the Model and Glossary (Version 16)

November 2008

A.0 Notation:

Time

$t \in I_T$ $\quad I_T = \{1,2,\ldots,T,\ldots\}$

Industry/Producer

$j \in I_{IND}$ $\quad I_{IND} = \{1,2,\ldots,35\}$

Legal form of organization

$c \in I_{LEGAL}$ $\quad$ {corporate, non-corporate}

IO Commodities

$i \in I_{COM}$ $\quad I_{COM} = \{1,2,\ldots,35\}$

Industry Inputs

$i \in I_{INP}$ $\quad I_{INP} = \{1,2,\ldots,35,NCI,K,L\}$

NIPA PCE Commodities

$n \in I_{PCE}$ $\quad I_{PCE} = \{1,2,\ldots,38\}$

Purchasers of domestic output

$j \in I_{BUY}$ $\quad I_{BUY} = \{1,2,\ldots,35,C,I,G,X\}$

Households

$k \in I_{POP}$

Nodes of production function

$m \in I_{PNODE}$ $\quad I_{PNODE} = \{EN,M,\ldots,WP\}$

$i \in I_{PNODEm}$ $\quad I_{PNODEm}$ in Table 2.4

Nodes of consumption function

$m \in I_{CNODE}$ $\quad I_{CNODE} = \{ND,EN,\ldots,RC\}$

$i \in I_{CNODEm}$ $\quad I_{CNODEm}$ in Table 2.3

Nodes of investment function

$m \in I_{INV}$ $\quad I_{INV} = \{fixed, \ldots, mining\}$

$i \in I_{INVm}$ $\quad I_{INVm}$

Nodes of capital demand function

$s \in I_{ASSET}$ $\quad I_{ASSET} = \{\text{short-lived,long-lived}\}$

Levels of government

$f \in I_{\text{GOV}}$ $\quad I_{\text{GOV}} = \{\text{federal,state \& local}\}$

Externalities

$x \in I_{EXT}$ $\quad I_{\text{EXT}} = \{1, 2, 3, 4\} = \{CO_2, SO_2, \ldots\}$

Vector of *1*'s

ι

Transpose of matrix *A*

A'

Diagonal matrix of a vector *v*

Diag(*v*)

Exogenous variables in blue

A.1 Household Sector

***Household first stage decision, Euler equation*:**

$$\text{Max}\sum_{t=1}^{\infty}\frac{N_t^{eq}}{(1+\rho)^t}\left(F_t/N_t^{eq}\right)^{1-\frac{1}{\sigma}} \qquad \text{given } K_0,\{\bar{L}_t\}, \tag{A.1.1}$$

subject to

$$WF \equiv PK_0K_0 + BG_0 + BF_0 + \sum_{t=1}^{\infty}\frac{Y_t^{full}}{\prod_{s=1}^{t}1+r_s} \geq \sum_{t=1}^{\infty}\frac{P_t^F F_t}{\prod_{s=1}^{t}1+r_s} \tag{A.1.2}$$

$$Y^{full} = P^h\bar{L} + G^{TRAN} - twW_{t-1} - TLUMP - H^{row} - R^N \tag{A.1.3}$$

$$\left[\frac{F_t/N_t^{\text{eq}}}{F_{t-1}/N_{t-1}^{\text{eq}}}\right]^{1/\sigma} = \frac{1+r_t}{1+\rho}\frac{PF_{t-1}}{PF_t} \tag{A.1.4}$$

Wealth, private income and savings:

$$W_t \equiv PK_tK_t + BG_t + BF_t \tag{A.1.5}$$

$$YF_t = YK_t^{net} + P^h\bar{L} + G^{tran} - TLUMP_t - twW_{t-1} + G_t^{Ktran} + R\_CON^{reb} \tag{A.1.6}$$

$$\begin{aligned} Y_t &= YK_t^{net} + YL_t + G^{tran} - TLUMP_t - twW_{t-1} + G_t^{Ktran} + R\_CON^{reb} \\ &= YF_t - p_t^{leis}L_t^{leis} = YF_t - w_t\psi_C^R C_R^N \end{aligned} \tag{A.1.7}$$

$$YL = P^hLS\frac{1\text{-}tl^a}{1\text{-}tl^m} = (1-tl^a)YL^{gross} = (1-tl^a)\sum_j PLD_jLD_j \tag{A.1.8}$$

YK^{net} is eq. A.3.15

$$\begin{aligned} S_t &= YF_t - P_t^F F_t - H_t^{row} - R_t^N - R\_ITC \\ &= YF_t - w_t\psi_C^R C_R^N - P_t^C C_t - H_t^{row} - R_t^N - R\_ITC \end{aligned} \tag{A.1.9}$$

$$= Y_t - P_t^C C_t - H_t^{row} - R_t^N - R\_ITC \tag{A.1.10}$$

H^{row} household and business net transfers to foreigners (C913+C913b)

***Household second stage decision, goods and leisure choice*:**

Rank 2 model estimated for household k from CEX data:

$$\ln V_k = \alpha_0 + \alpha^H \ln \frac{\rho_r}{m_k} + \tfrac{1}{2} \ln \frac{\rho_r}{m_k}' \mathbf{B}^H \ln \frac{\rho_r}{m_k} + \ln \frac{\rho_r}{m_k}' \mathbf{B}_{pA} A_k \qquad \text{(A.1.11)}$$

$\mathbf{c}_k^X = (C_{NDk}^X, C_{Kk}^X, C_{CSk}^X, C_{Rk}^X)'$ consumption vector

$\boldsymbol{\rho}_r = (p_{ND}^r, p_K^r, p_{SV}^r, p_R^r)'$ price vector indexed by region

$$m_k = p_{ND}^r C_{NDk}^X + p_K^r C_{LKk}^X, p_{CS}^r C_{CSk}^X + p_R^r C_{Rk}^X \qquad \text{(A.1.12)}$$

$$C_{Rk}^X = \sum_{m\ adults} q_{kt}^m (5110 - hoursworked_{kt}^m); \quad q_{kt}^m = p_{Rt}^m / p_{Rt}^r \qquad \text{(A.1.13)}$$

A_k = (0,1) dummies for
k= {1 child, 2 children, 3+children, 1 aged18-64, 2 aged18-26, 3 aged18-64, 1elderly, 2+elderly, MidWest, South, West, nonwhite, female, rural}
(left out groups: 0child, 0aged18-64, 0elderly, Northeast, white, male, urban)

Above econometric model modified for use in IGEM. Shares on CEX basis:

$$SC^X = \frac{\alpha^H + B^H \ln P^{H1} - B^H \iota \xi^d + B_{pA} \xi^L}{D(p)} \qquad \text{(A.1.14)}$$

$$SC^X \equiv \left(\frac{PC_{ND}^X C_{ND}^X}{MF^X}, \frac{PC_K^X C_K^X}{MF^X}, \frac{PC_{CS}^X C_{CS}^X}{MF^X}, \frac{PC_R^X C_R^X}{MF^X} \right),$$

$$D(p) = -1 + \iota' B^H \ln P^{H1} \qquad \text{(A.1.15)}$$

$$\ln P^{H1} = (\ln PC_{ND}^X, \ln PC_K^X, \ln PC_{CS}^X, \ln PC_R^X)$$

$$\xi_j^L = (\xi_{1child}^L, \ldots, \xi_{rural}^L)' = \sum_{k \in j} m_k / M = \sum_{k \in j} \lambda_{kt} \qquad \text{j={1 child,….,female,rural}} \qquad \text{(A.1.16)}$$

$$\xi^d = \sum_k M_k \ln M_k / M = \xi^{dd} + \ln PFF; \qquad \text{(A.1.17)}$$

$$\xi_t^{dd} = \sum_k \lambda_{kt} \ln \lambda_{kt}; \qquad \lambda_{kt} = nf_{kt} \frac{m_{k,baseyear}}{M_{baseyear}} \qquad \text{(A.1.18)}$$

Exogenous bridge equation between CEX units and NIPA units:

$$\Delta SC_{it} = SC_{it}^N - SC_{it}^X \qquad \text{i={ND,K,CS,R}} \qquad \text{(A.1.19)}$$

$$\Delta SC_{it} = \alpha + \beta \Delta SC_{it} + \varepsilon_{it} \qquad \varepsilon_{it} = \rho \varepsilon_{it} + u_{it} \qquad \text{(A.1.20)}$$

$$SC^N \equiv \left(\frac{PN^{ND} N^{ND}}{MF^N}, \frac{PN^K N^K}{MF^N}, \frac{PN^{CS} N^{CS}}{MF^N}, \frac{PN^R N^R}{MF^N} \right),$$

$PC^X_{ND} = PN^{ND}$ (no bridge for prices)

$PC^X_{CS} = PN^{CS}$ (A.1.21)

$PC^X_K = PN^K = PKD_C$

$PC^X_R = PN^R = \psi^R_C P^h$

$MF^N = PF * F = PCC.CC + PN^R N^R$ (A.1.22)

$VCC = PCC.CC = PN^{ND} N^{ND} + PN^K N^K + PN^{CS} N^{CS}$ (A.1.23)

Time endowment, labor supply, leisure, price of hours, price of leisure:

$P^h \bar{L} = P^h LS + PN^R N^R$ (A.1.24)

$LS = \bar{L} - \psi^R_C N^R$ (A.1.25)

$d \ln \bar{L}_t = \sum_k \frac{1}{2} (v^L_{kt} + v^L_{kt-1}) d \ln(14 * 365 * POP_{kt})$; $\quad v^L_{kt} = (1 - tl^m_t) P^L_{kt}$ (A.1.26)

Household third stage decision, allocation of detailed PCE:

NESTED STRUCTURE OF CONSUMPTION (A.1.27)

1	$F = F(N^{ND}, N_K, N^{CS}, N_R)$	Aggregate Full consumption
2	$N^{ND} = N^{ND}(N^{EN}, N^{F}, N^{CG})$	Nondurables
3	$N^{EN} = N^{EN}(N_6, N^{FC}, N_{18}, N_{19})$	Energy
4	$N^{F} = N^{F}(N_1, N_2, N_3, N_9)$	Food
5	$N^{CG} = N^{CG}(N^{CL}, N^{HA}, N_{12}, N^{MS})$	Consumer goods
6	$N^{CS} = N^{CS}(N^{H}, N^{HO}, N^{TR}, N^{MD}, N^{MI})$	Consumer services
7	$N^{FC} = N^{FC}(N_7, N_8)$	Fuel and coal
8	$N^{CL} = N^{CL}(N_4, N_5)$	Clothing and shoes
9	$N^{HA} = N^{HA}(N_{10}, N_{11})$	Household articles
10	$N^{MS} = N^{MS}(N_{13}, N_{14}, N_{15}, N_{16})$	Miscellaneous nondurables
11	$N^{H} = N^{H}(N_{17}, N_{34})$	Housing services (rental, maintenance)
12	$N^{HO} = N^{HO}(N_{20}, N_{21}, N_{22}, N_{23})$	Household operation
13	$N^{TR} = N^{TR}(N_{24}, N_{25})$	Transportation
14	$N^{MD} = N^{MD}(N_{26}, N_{27})$	Medical
15	$N^{MI} = N^{MI}(N_{28}, N^{BU}, N^{RC}, N_{32})$	Miscellaneous services
16	$N^{BU} = N^{BU}(N_{29}, N_{30})$	Business services
17	$N^{RC} = N^{RC}(N_{31}, N_{33})$	Recreation
	subscripts $\in I_{PCE}$	

Price dual of **lower** tiers consumption demands $N^m(\ldots)$:

$$\ln PN^m = \alpha^{Hm}{}' \ln P^{Hm} + \frac{1}{2} \ln P^{Hm}{}' B^{Hm} \ln P^{Hm} + \ln P^{Hm}{}' f^{Hm} \qquad m \in I_{CNODE} \qquad \text{(A.1.28)}$$

$$\ln P^{Hm} \equiv (\ln PN_{m1}, \ldots, \ln PN_{mi}, \ldots, \ln PN_{m,im})' \qquad i \in I_{CNODEm} \qquad \text{(A.1.29)}$$

$$f_t^{Hm} = F^{Hm} f_{t-1}^{Hm} + v_t^{Hm} \qquad \text{(A.1.30)}$$

$$SN^m = \begin{bmatrix} PN_{m1} N_{m1} / PN^m N^m \\ \cdots \\ PN_{m,im} N_{m,im} / PN^m N^m \end{bmatrix} = \alpha^{Hm} + B^{Hm} \ln PN^{Hm} + f^{Hm} \qquad \text{(A.1.31)}$$

$$PN_{mi} \in \left\{ PN_1, \ldots, PN_{34}, PN^{ND}, \ldots, PN^{RC} \right\}$$
$$N_{mi} \in \left\{ N_1, \ldots, N_{34}, N^{ND}, \ldots, N^{RC} \right\}$$

$$\begin{aligned} PN_1N_1 &= s_1^{con} PF.F = SN_1^F * SN_2^{ND} * SN_1^{\text{TOP}} * PF.F \\ PN_2N_2 &= s_2^{con} PF.F = SN_2^F * SN_2^{ND} * SN_1^{\text{TOP}} * PF.F \\ &\cdots \\ PN_{34}N_{34} &= s_{34}^{con} PF.F = SN_2^H * SN_1^{CS} * SN_3^{\text{TOP}} * PF.F \end{aligned} \qquad \text{(A.1.32)}$$

$$VN \equiv \left(PN_1N_1, \ldots, PN_{34}N_{34} PKD_C KD_C \right)$$

$PN = \mathbf{H}' PS^C$ where the components of PS^C : (A.1.33)

$PS_i^C = (1 + tc_i) PS_i \qquad i \in I_{COM}$ (A.1.34)

$PS_N^C = (1 + tc_N) PNCI_C$

$PS_K^C = (1 + tc_K) PKD_C$ (A.1.35)

$PS_L^C = (1 + tc_L) PLD_C$

Converting from NIPA categories to IO categories:

$$\begin{aligned} VC &\equiv \left(PS_1C_1, \ldots, PS_{35}C_{35}, \ldots, PLD_C LD_C \right)' \\ &= \mathbf{H} VN \end{aligned} \qquad \text{(A.1.36)}$$

$C_i = VC_i / PS_i \qquad i \in I_{INP}$ (A.1.37)

$$C^P \equiv \left(C_1, C_2, \ldots, C_{35} \right)'$$
$$C \equiv \left(C_1, \ldots, C_{35}, NCI_C, KD_C, LD_C \right)'$$

Simple Cobb-Douglas price index of consumption:

$$\ln PCC = \sum_{i=1}^{I_{INP}} \frac{VC_i}{VCC} \log PS_i \qquad \text{(A.1.38)}$$

$CC = VCC / PCC$ (A.1.39)

A.2 Producer Model

NESTED STRUCTURE OF PRODUCTION (A.2.1)

$QI_j = QI^j(KD_j, LD_j, QP^{jE}, QP^{jM})$ Industry output

$QP^{jE} = QP^E(QP_3^j, QP_4^j, QP_{16}^j, QP_{30}^j, QP_{31}^j)$ Energy aggregate

$QP^{jM} = QP^M(QP_6^j, QP^{jMA}, QP^{jME}, QP^{jMN}, QP^{jMS})$ Material aggregate

$QP^{jMA} = QP^{AG}(QP_1^j, QP_7^j, QP_8^j, QP^{jTX}, QP^{jWP})$ Agriculture Materials

$QP^{jMM} = QP^{MM}(QP^{jFM}, QP^{jMC}, QP^{jEQ})$ Metallic Materials

$QP^{jMN} = QP^{MN}(QP_5^j, QP_{15}^j, QP_{17}^j, QP_{19}^j, QP_{27}^j)$ Non-metallic Materials

$QP^{jMS} = QP^{MS}(QP_{28}^j, QP_{32}^j, QP_{33}^j, QP_{34}^j, QP^{jOS})$ Service Materials

$QP^{jTA} = QP^{TA}(QP_9^j, QP_{10}^j, QP_{18}^j)$ Textile-Apparel

$QP^{jWP} = QP^{WP}(QP_{11}^j, QP_{12}^j, QP_{13}^j, QP_{14}^j)$ Wood and paper aggregate

$QP^{jOS} = QP^{OS}(QP_{29}^j, QP_{35}^j, QP_{NCI}^j)$ Miscellaneous services

$QP^{jFM} = QP^{FM}(QP_2^j, QP_{20}^j, QP_{21}^j)$ Fabricated-Other metal

$QP^{jMC} = QP^{MC}(QP_{22}^j, QP_{23}^j)$ Machinery aggregate

$QP^{jEQ} = QP^{EQ}(QP_{24}^j, QP_{25}^j, QP_{26}^j)$ Equipment aggregate

Price dual of **top** tier of production function $QI_j = QI(\ldots)$:

$$PO_j = PO^j(PKD_j, PLD_j, PP^{jE}, PP^{jM}, t; \lambda_j, A^{agg}) \qquad j \in I_{IND}$$

$A_t^{agg} = (1 - \Delta A^{agg}) A_{t-1}^{agg}$ (A.2.2)

λ_j exogenous productivity shock in industry j

ΔA^{agg} exogenous aggregate productivity shock, common to all

option "logistic_gt":

$$\ln PO_j = \alpha_0^j + \alpha^{Pj'} \ln P^{Pj0} + \frac{1}{2} \ln P^{Pj0'} B^{Pj} \ln P^{Pj0} + \ln P^{Pj0'} B_{pt}^j g(t) + \alpha_t^j g(t) + \frac{1}{2} \beta_{tt}^j g(t)^2 + \ln \lambda_j + \ln A^{agg} \quad \text{(A.2.3)}$$

where $g(t) = \dfrac{1}{1 + \exp(-\mu^j (t - \tau^j))}$. (A.2.4)

$$A_j^{TFP} = \ln P^{Pj0'} B_{pt}^j g(t) + \alpha_t^j g(t) + \frac{1}{2} \beta_{tt}^j g(t)^2 + \ln \lambda_j + \ln A^{agg} \quad \text{(A.2.5)}$$

$$\ln P^{Pj0} \equiv (\ln PKD_j, \ln PLD_j, \ln PP^{jEN}, \ln PP^{jM})'$$

$$SP^{jTOP} = \begin{bmatrix} PKD_j KD_j / PQ_j QI_j \\ \dots \\ PP^{jM} QP^{jM} / PO_j QI_j \end{bmatrix} = \alpha^{Pj} + B^{Pj} \ln P^{Pj0} + B_{pt}^{j} g(t) \qquad \text{(A.2.6)}$$

option "kalman":

$$\ln PO_j = \alpha_0^j + \alpha^{Pj'} \ln P^{Pj0} + \frac{1}{2} \ln P^{Pj0'} B^{Pj} \ln P^{Pj0} + \ln P^{Pj0'} f_t^{Pj} + f_t^{j} + \ln \lambda_j + \ln A^{agg} \qquad \text{(A.2.7)}$$

$$\xi_t^{Pj} = F^{Pj} \xi_{t-1}^{Pj} + v_t^{Pj} \qquad \text{(A.2.8)}$$

$$\xi_t^{Pj} = (1, f_{Kt}^{Pj}, f_{Lt}^{Pj}, f_{Et}^{Pj}, f_{Mt}^{Pj}, \Delta f_t^{j})'$$

$$A_j^{TFP} = \ln P^{Pj0'} f_t^{Pj} + f_t^{j} + \ln \lambda_j + \ln A^{agg} \qquad \text{(A.2.9)}$$

$$SP^{jTOP} = \begin{bmatrix} s_j^K \\ s_j^L \\ s_j^E \\ s_j^M \\ \\ \end{bmatrix} = \begin{bmatrix} PKD_j KD_j / PQ_j QI_j \\ \dots \\ PP^{jM} QP^{jM} / PO_j QI_j \end{bmatrix} = \alpha^{Pj} + B^{Pj} \ln P^{Pj0} + f_t^{Pj} \qquad \text{(A.2.10)}$$

Price dual of **lower** tiers of production functions $QP^{jm} = QP(\dots)$:

$$\ln P^{Pjm} \equiv (\ln PP_{m1}^{j}, \dots, \ln PP_{mi}^{j}, \dots, \ln PP_{m,im}^{j})' \qquad i \in I_{PNODEm}$$

option "lowertier_std":

$$\ln PP^{jm} = \alpha_0^{jm} + \alpha^{Pjm'} \ln P^{Pjm} + \frac{1}{2} \ln P^{Pjm'} B^{Pjm} \ln P^{Pjm} \qquad m \in I_{PNODE} \qquad \text{(A.2.11)}$$

$$SP^{jm} = \begin{bmatrix} PP_{m1}^{j} QP_{m1}^{j} / PQ^{jm} QP^{jm} \\ \dots \\ PP_{m,im}^{j} QP_{m,im}^{j} / PP^{jm} QP^{jm} \end{bmatrix} = \alpha^{Pjm} + B^{Pjm} \ln P^{Pjm} \qquad \text{(A.2.12)}$$

option "lowertier_kalman":

$$\ln PP^{jm} = \alpha_0^{jm} + \alpha^{Pjm'} \ln P^{Pjm} + \frac{1}{2} \ln P^{Pjm'} B^{Pjm} \ln P^{Pjm} + \ln P^{Pjm'} f_t^{Pjm} \qquad m \in I_{PNODE} \qquad \text{(A.2.13)}$$

$$f_t^{Pjm} = F^{Pjm} f_{t-1}^{Pjm} + v_t^{Pjm} \qquad \text{(A.2.14)}$$

$$SP^{jm} = \begin{bmatrix} PP_{m1}^{j} QP_{m1}^{j} / PQ^{jm} QP^{jm} \\ \dots \\ PP_{m,im}^{j} QP_{m,im}^{j} / PP^{jm} QP^{jm} \end{bmatrix} = \alpha^{Pjm} + B^{Pjm} \ln P^{Pjm} + f_t^{Pjm} \qquad \text{(A.2.15)}$$

$$PP_{mi}^{j} \in \{PS_1, \dots, PS_{35}, PNCI_j, PP^{jMA}, \dots, PP^{jOS}\}$$

$$QP_{mi}^{j} \in \left\{QP_1^{j},\ldots,QP_{35}^{j},NCI_j,QP^{jMA},\ldots,QP^{jOS}\right\}$$

Vectors for use in formulas below:

$$V^{QI} \equiv \left(PO_1QI_1,\ldots,PO_{35}QI_{35}\right)'$$

Taxes (net vs. gross output):

$$\begin{aligned} PI_j &= \left(1+tt_j+tx_j^{v}\right)PO_j+tu_j+tx_j^{u} \qquad j \in I_{\text{IND}} \\ &= \left(1+tt_j^{full}\right)PO_j \end{aligned} \tag{A.2.16}$$

$$\begin{aligned} VT^{QI} &\equiv \left(PI_1QI_1,\ldots,PI_{35}QI_{35}\right)' \\ &= Diag(\iota+tt^{full})V^{QI} \end{aligned} \tag{A.2.17}$$

Commodities from industry outputs:

$\mathbf{M}=[M_{ji}]$ = value of commodity i made by industry j (A.2.18)

$$m_{ji}^{col}=\frac{M_{ji}}{\sum_k M_{ki}}; \qquad m_{ji}^{row}=\frac{M_{ji}}{\sum_k M_{jk}} \tag{A.2.19}$$

$$PC=\mathbf{m}^{\mathbf{col}\,\prime}PI \tag{A.2.20}$$

$$\begin{aligned} V^{QC} &\equiv \left(PC_1QC_1,\ldots,PC_{35}QC_{35}\right)' \\ &= \mathbf{m}^{\mathbf{row}\,\prime}VT^{QI} \end{aligned} \tag{A.2.21}$$

$$QC_i = V^{QC}{}_i / PC_i \qquad i \in I_{\text{COM}} \tag{A.2.22}$$

The input-output USE matrix, in share terms, used in eq. A.6.4***:***

$$\begin{aligned} A_{1j} &= SP_1^{jAG} * SP_2^{jM} * SP_4^{jTOP} \\ A_{2j} &= SP_1^{jFM} * SP_1^{jMM} * SP_3^{jM} * SP_4^{jTOP} \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{35j} &= SP_2^{jOS} * SP_5^{jMS} * SP_5^{jM} * SP_4^{jTOP} \end{aligned} \tag{A.2.23}$$

$$A_j \equiv \left(A_{1j},A_{2j},\ldots,A_{35j}\right)' \qquad \text{j} \in I_{\text{IND}} \tag{A.2.24}$$

$$\mathbf{A} \equiv [A_1,A_2,\ldots,A_{35}]' \tag{A.2.24b}$$

$$PS_i QP_i^j = A_{ij} VQI_j \quad \text{(A.2.25)}$$

$$PNCI_j NCI_j = SP_3^{jOS} * SP_5^{jMS} * SP_5^{jM} * SP_4^{jTOP} * PO_j * QI_j \quad \text{(A.2.26)}$$

$$PKD_j KD_j = SP_1^{jTOP} * PO_j * QI_j \quad \text{(A.2.27)}$$

$$PLD_j LD_j = SP_2^{jTOP} * PO_j * QI_j \quad \text{(A.2.28)}$$

For j=oil mining; by default:

$KD_4 = \overline{KD}_4$ (oil sector) (A.2.29)

PKD_4 independent endogenous variable

If set flag for “oil_kap_free” then

$$PKD_4 = \psi_4^K PKD \quad \text{(A.2.30)}$$

A.3 Capital and Investment

The Bank as owner of economy aggregate capital

$$\text{Max}\sum_{t=u}^{\infty}\frac{(1\text{-}tk)(PKD_t\psi^K K_{t\text{-}1}\text{-}tpPK_{t\text{-}1})\text{-}(1\text{-}t^{ITC})PII_t I_t^a}{\prod_{s=u}^{t}1+r_s} \tag{A.3.1}$$

subject to

$$K_t = (1-\delta)K_{t-1} + \psi^I \varepsilon^I I_t^a \tag{A.3.2}$$

ε_t^I investment productivity shock

Hamiltonian:

$$\frac{(1\text{-}tk)(PKD_t\psi^K K_{t\text{-}1}\text{-}tpPK_{t\text{-}1}\text{K}_{t\text{-}1})\text{-}(1-t^{ITC})PII_t I_t^a}{\prod_{s=u}^{t}1+r_s} + \frac{\lambda_t}{\prod_{s=u}^{t}1+r_s}\left((1-\delta)K_{t-1} + \psi^I \varepsilon^I I_t^a - K_t\right) \tag{A.3.3}$$

Euler equation:

$$(1+r_t)\frac{PII_{t-1}}{\psi_{t-1}^I \varepsilon_{t-1}^I} = \frac{1-tk}{1-t^{ITC}}(PKD_t\psi_t^K - tpPK_{t-1}) + (1-\delta)\frac{PII_t}{\psi_t^I \varepsilon_t^I} \tag{A.3.4}$$

Aggregation relationships:

$$PK_t = \psi_t^{PK} PII_t (1-t^{ITC}) \tag{A.3.5}$$

$$KD_t = \psi_t^K K_{t-1} \tag{A.3.6}$$

In the projection period:

$$\psi_t^K = \psi_{t-1}^K + \Delta\psi_t^K \tag{A.3.7}$$

$$\Delta\psi_t^K = \alpha_0^{\psi K} + \alpha_1^{\psi K}\Delta\psi_{t-1}^K + \alpha_2^{\psi K}\Delta\psi_{t-2}^K + v_t^{\psi K} \tag{A.3.8a}$$

$$\Delta\psi_t^I = \alpha_0^{\psi I} + \alpha_1^{\psi I}\Delta\psi_{t-1}^I + \alpha_2^{\psi I}\Delta\psi_{t-2}^I + v_t^{\psi I} \tag{A.3.8b}$$

$$\Delta\psi_t^{PK} = \alpha_0^{PK} + \alpha_1^{PK}\Delta\psi_{t-1}^{PK} + \alpha_2^{PK}\Delta\psi_{t-2}^{PK} + v_t^{PK} \tag{A.3.8c}$$

$$VII = PII.I^a \tag{A.3.9}$$

Standard version with no corporate-noncorp distinction ("version 8"):

$$PK^{gain} = \left(\frac{PK_t - PK_{t-1}}{PK_{t-1}} - \delta\right) \quad \text{(A.3.10)}$$

$$VK^{gain} = \left(\frac{PK_t - PK_{t-1}}{PK_{t-1}} - \delta\right) PK_{t-1}K_{t-1} \quad \text{(A.3.11)}$$

$$Y^I = r(BG + BF) \quad \text{(A.3.12)}$$

$$YK^{gov} = (1-tk)PKD_{35}KD_{35} \quad \text{(A.3.13)}$$

$$YK = \sum_{j=1}^{36} PKD_j KD_j - R\_K^{hh} \quad \text{(A.3.14)}$$

$$YK_t^{net} = DIV - YK^{gov} + (1-tk)\left(GINT_t^p + Y^{row}\right) \quad \text{(A.3.15)}$$

$$DIV = (1-tk)YK - tpPK_tK_t \quad \text{(A.3.16)}$$

$$\alpha^{div} = DIV / PK_{t-1}K_{t-1} \quad \text{(A.3.16b)}$$

$$r_t = \alpha^{div} + PK^{gain} \quad \text{(A.3.17)}$$

NESTED STRUCTURE OF INVESTMENT

$I^{a} = I^{a}(I^{\text{fixed}}, I^{\text{inventory}})$	Aggregate investment
$I^{\text{fixed}} = I^{FX}(I^{\text{long}}, I^{\text{short}})$	Fixed investment aggregate
$I^{\text{inventory}} = I^{IY}$	Change in business inventories
$I^{\text{long}} = I^{LG}(I_{6}, I_{33})$	Long-lived investment aggregate
$I^{\text{short}} = I^{SH}(I^{\text{vehicles}}, I^{\text{machinery}}, I^{\text{services}})$	Short-lived investment aggregate
$I^{\text{vehicles}} = I^{VE}(I_{24}, I_{25})$	Vehicle aggregate
$I^{\text{machinery}} = I^{MC}(I_{22}, I_{23}, I^{\text{other}-m})$	Machinery aggregate
$I^{\text{services}} = I^{SV}(I_{32}, I^{\text{other}-s})$	Service aggregate
$I^{\text{other}-m} = I^{MO}(I^{\text{gadgets}}, I^{\text{wood}}, I^{\text{nonmetal}}, I^{\text{misc}})$	Other machinery aggregate
$I^{\text{other}-s} = I^{SO}(I_{34}, I^{\text{movers}})$	Other services aggregate
$I^{\text{gadgets}} = I^{GD}(I_{20}, I_{21}, I_{26})$	Metals and instruments aggregate
$I^{\text{wood}} = I^{WD}(I_{11}, I_{12})$	Wood products aggregate
$I^{\text{nonmetal}} = I^{MN}(I_{15}, I_{17}, I_{29}, I_{27})$	Nonmetallic products aggregate
$I^{\text{misc}} = I^{OO}(I^{\text{textile}}, I_{13}, I^{\text{mining}})$	Miscellaneous aggregate
$I^{\text{mover}} = I^{TC}(I_{28}, I^{29})$	Transportation and Communicating aggregate
$I^{\text{textile}} = I^{TX}(I_{9}, I_{10}, I_{18}, I_{NCI})$	Textile aggregate
$I^{\text{mining}} = I^{MG}(I_{2}, I_{4})$	Minerals aggregate

At **top** tier of investment functions $I = I(\ldots)$:

$$VII = VII^{\text{fixed}} + VII^{\text{invy}} \tag{A.3.18}$$

$$\frac{VII^{\text{invy}}}{VII} = \alpha^{IY} \tag{A.3.19}$$

$$VII_{\text{i}}^{\text{invy}} = \alpha_{i}^{IY} VII^{\text{invy}} \qquad i \in I_{\text{COM}} \tag{A.3.20}$$

Price dual of fixed investment demand tiers $I^{m} = I^{m}(\ldots)$:

$$\ln PII^{m} = \alpha^{Im} \ln P^{Im} + \tfrac{1}{2} \ln P^{Im\,\prime} B^{Im} \ln P^{Im} + \ln P^{Im\,\prime} f_{t}^{Im} + \log \lambda^{I} \qquad \text{m} \in \text{I}_{\text{INV}} \tag{A.3.21}$$

$$f_{t}^{Im} = F^{Im} f_{t-1}^{Im} + v_{t}^{Im} \tag{A.3.22}$$

$$\ln P^{Im} \equiv \left(\ln PII_{m1}, \ldots, \ln PII_{mi}, \ldots, \ln PII_{m,im}\right) \quad i \in I_{INVm}$$

$$SI^{\mathrm{m}} = \begin{bmatrix} PII_{m1} I^{f}_{m1} / PII^{m} I^{m} \\ \dots \\ PII_{m,im} I^{f}_{m,im} / PII^{m} I^{m} \end{bmatrix} = \alpha^{Im} + B^{Im} \ln PII^{Im} + f_t^{Im} \qquad \begin{matrix} \mathrm{m} \in \mathrm{I_{INV}} \\ \mathrm{mi} \in \mathrm{I_{INVm}} \end{matrix} \qquad \text{(A.3.23)}$$

$$PII_{mi} \in \left\{ PS_1, \dots, PS_{35}, PII^{\mathrm{fixed}}, \dots, PII^{\mathrm{mining}} \right\}$$
$$I_{mi} \in \left\{ I^{f}_1, \dots, I^{f}_{35}, I^{\mathrm{fixed}}, \dots, I^{\mathrm{mining}} \right\}$$

Share demands under Cobb-Douglas option:

$$SI = \begin{bmatrix} PS_1 I^{f}_1 / VII \\ \dots \\ PS_{NINP} I^{f}_{NINP} / VII \end{bmatrix} = \alpha^{CD,Im} \qquad \text{(A.3.23b)}$$

Values of individual commodities making up aggregate investment demand:

$$VI_i = VI_i^{fixed} + VI_i^{inventory}:$$

$$VI_1 = 0 + VI_1^{\mathrm{invy}}$$
$$VI_2 = SI_1^{MG} * SI_3^{OO} * SI_4^{MO} * SI_3^{MC} * SI_2^{SH} * SI_2^{FX} VII^{\mathrm{fixed}} + VI_2^{\mathrm{invy}}$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad \text{(A.3.24)}$$
$$VI_{34} = SI_1^{SO} * SI_2^{SV} * SI_3^{SH} * SI_2^{FX} VII^{\mathrm{fixed}} + VI_{343}^{\mathrm{invy}}$$
$$VI_{35} = 0$$

$$I_i = VI_i / PS_i \qquad \text{(A.3.25)}$$

vectors used in A.6.2:

$$VI \equiv \left(PS_1 I_1, \dots, PS_{35} I_{35}, PNCI_I NCI_I \right)'$$
$$I^P \equiv \left(I_1, \dots, I_{35} \right)'$$
$$I \equiv \left(I_1, \dots, I_{35}, NCI_1 \right)'$$

Disaggregated capital version (referred to as "version 9" of IGEM)

$$YK_t = \sum_{j-1}^{34} PKD_j KD_j + PKD_h KD_h + \left(i_t BF_{t-1} + Y_t^{row,adj} + i_t BG_{t-1} + GINT_t^{adj} \right) \quad \text{(A.3.26)}$$

cash_flow, CF_j is (A.3.64); BH is (A.3.67).

$$YK_t^{net} = \sum_{j-1}^{34} CF_j + PKD_h KD_h - t_h^e BH - t_h^p PII_t K_{ht-1} + (1-t_n^e)\left(i_t BF_{t-1} + Y_t^{row,adj} + i_t BG_{t-1} + GINT_t^{adj} \right) \quad \text{(A.3.27)}$$

Capital services

$$KD = KD_1 + \cdots + KD_{35} + KD_h \quad \text{(A.3.28)}$$

$$KD_j = KD^j(KD_{jc}, KD_{jn}) \qquad \text{corporate, noncorporate} \qquad j \in I_{IND} \quad \text{(A.3.29)}$$

$$KD_{jc} = KD(KD_{jcs}, KD_{jcl}) \qquad \text{short, long-lived assets} \qquad c = c,n \quad \text{(A.3.30)}$$

$$VKD_{jcst} = PKD_{jcst} KD_{jcst} \qquad \text{value of capital services} \quad \text{(A.3.31)}$$

****total Industry capital***

$$\ln PKD_j = \alpha_{KD0}^j + \alpha_{KD}^j \ln P + \frac{1}{2} \ln P' B_{KD}^j \ln P, \qquad \ln P = (\ln PKD_{jc}, \ln PKD_{jn})' \quad \text{(A.3.32)}$$

$$\begin{bmatrix} PKD_{jc} KD_{jc} / VKD_j \\ PKD_{jn} KD_{jn} / VKD_j \end{bmatrix} = \alpha_{KD}^j + B \ln P, \quad \text{(A.3.33)}$$

****Corporate capital***

$$\ln PKD_{jc} = \alpha_{KD0}^c + \alpha_{KD}^{jc} \ln P + \frac{1}{2} \ln P' B_{KD}^{jc} \ln P, \qquad \ln P = (\ln PKD_{jcs}, \ln PKD_{jnl})' \quad \text{(A.3.34)}$$

$$\begin{bmatrix} PKD_{jcs} KD_{jcs} / VKD_j \\ PKD_{jcl} KD_{jcl} / VKD_j \end{bmatrix} = \alpha_{KD}^{jc} + B_{KD}^{jc} \ln P, \quad \text{(A.3.35)}$$

$$PKD_{jcst} = \left[\frac{1 - t_{cs}^{ITC} - t_c z_{cs}}{1 - t_c} (r_{jc} + (1+\pi)\delta_{cs}) + \gamma_c^p t_c^p \right] PK_{t-1} \qquad \text{s=s,l} \quad \text{(A.3.36)}$$

where $r_{csj}^{net} = r_{jc} + (1+\pi)\delta_{cs}$ (A.3.37)

Note: We do not index PK by PK_{jcs} or PK_{jns}, but have a common price of capital goods for all buyers. Similarly, π is common to all equations

$$r_{jc} = (1-\beta_{jc})r_{ce}^{equ} + \beta_{jc}([1-(1-\gamma_c^i)t_c]i_t - \pi) \quad \text{(A.3.38)}$$

where $$r_c^{equ} = \frac{\rho^e - \pi\left[1-\left(1-\gamma_c^g\right)t_c^g\right]}{1-t_c^{earn}}\left(1-\alpha^{DIV}\gamma_c^d t_c\right) \quad \text{(A.3.38b)}$$

where $$t_c^{\text{earn}} = \alpha^{\text{DIV}} t_c^e + \left(1-\alpha^{\text{DIV}}\right)t_c^g \quad \text{(A.3.38c)}$$

Noncorporate capital

$$\ln PKD_{jn} = \alpha_{KD0}^{jn} + \alpha_{KD}^{jn}\ln P + \frac{1}{2}\ln P' B_{KD}^{jn}\ln P \quad \text{(A.3.39)}$$

$$\ln P = (\ln PKD_{jns}, \ln PKD_{jnl})'$$

$$PKD_{jns} = \left[\frac{1-t_{ns}^{ITC}-t_n^e z_{ns}}{1-t_n^e}(r_{jn}+(1+\pi)\delta_{ns}) + \gamma_n t_n^p\right]PK_{t-1} \qquad \text{s=s,l} \quad \text{(A.3.40)}$$

where $r_{nsj}^{net} = r_{jn} + (1+\pi)\delta_{ns}$ (A.3.40b)

$$\begin{bmatrix} PKD_{ns}KD_{ns}/VND_j \\ PKD_{nl}KD_{nl}/VND_j \end{bmatrix} = \alpha_{KD}^{jn} + B_{KD}^{jn}\ln P, \quad \text{(A.3.41)}$$

$$r_{jn} = \left(1-\beta_{jn}\right)\left(r_n^{equ}\right) + \beta_{jn}\left(1-\left(1-\gamma_n^i\right)t_n^e\right)i_t - \pi \quad \text{(A.3.42)}$$

$$r_n^{equ} = \rho^e - \pi\left[1-\left(1-\gamma_n^g\right)t_n^g\right] \quad \text{(A.3.42b)}$$

Household capital

$$PKD_{hs} = \left[r_h + (1+\pi)\delta_{hs} + \left(1-\gamma_h^p\right)t_{hs}^p\right]PK_{t-1} \qquad s = s,l \quad \text{(A.3.43)}$$

where $r_{hs}^{net} = r_h + (1+\pi)\delta_{hs}$ (A.3.44)

$$\ln PKD_h = \alpha_{KD0}^h + \alpha_{KD}^h\ln P + \frac{1}{2}\ln P' B_{KD}^h \ln P, \quad \text{(A.3.45)}$$

$$\ln P = (\ln PKD_{hs}, \ln PKD_{hl})'$$

$$\begin{bmatrix} PKD_{hs}KD_{hs}/VHD \\ PKD_{hl}KD_{hl}/VHD \end{bmatrix} = \alpha_{KD}^h + B_{KD}^h\ln P, \quad \text{(A.3.46)}$$

$$r_h = (1-\beta_h) r_h^{equ} + \beta_h [1-(1-\gamma_h^i) t_h^e (1-dhi)] i_t - \pi \quad \text{(A.3.47)}$$

where $r_h^{equ} = \rho^e - \pi\left[1-(1-\gamma_n^g) t_h^g\right]$ (A.3.48)

*Short, long

In data construction, short and long capital is aggregated over the corresponding components from the list of 51 BEA assets classes: $K_{jcst} = \sum_i K_{ijcst}$ and $K_{jclt} = \sum_i K_{ijclt}$ for $j = 1,\ldots,35$ and $c = c,l,h$.

In this version of the model we do not keep track of $K_{ijcst} = (1-\delta_i)K_{ijcst-1} + I_{ijcst}$. In other words there is no effort to link the investment column of the IO Table (A.3.17) and I_{ijcst}.

$$t_c = t_c^f (1-t_c^s) + t_c^s \quad \text{(A.3.50)}$$

Replacement cost

$$VK_j^{rep} = \sum_{cs} (1-t_{cs}^{ITC} - t_c z_{cs}) PK_{t-1} K_{jcst-1} \quad \text{(A.3.51)}$$

Value of depreciation deductions and property tax

$$D_{jc} = \sum_{s,l} z_{cs} (r_c + (1+\pi)\delta_{cs})(1-t_{cs}^{ITC} - t_c z_{cs}) PK_{t-1} K_{jcst-1} \quad \text{(A3.52)}$$

$$D_{jn} = \sum_{s,l} z_{ns} (r_n + (1+\pi)\delta_{ns})(1-t_{ns}^{ITC} - t_n z_{ns}) PK_{t-1} K_{jnst-1} \quad \text{(A.3.53)}$$

$$R_{jc}^p = t_c^p (1-t_{cs}^{ITC} - t_c z_{cs}) PK_{t-1} K_{jcst-1} \qquad c = c,n \quad j = 1 \in I_{IND} \quad \text{(A.3.54)}$$

interest cost

$$IC_{cj} = \beta_{jc} \sum_{s,l} (1-t_{cs}^{ITC} - t_c z_{cs}) PK_{t-1} K_{jcst-1} i_t \qquad c = c,n \quad j = 1 \in I_{IND} \quad \text{(A.3.55)}$$

Corporate income tax base; corp tax revenue

$$BQ_j = PKD_{jc} KD_{jc} - D_{jc} - IC_{jc} - R_{jc}^p \quad \text{(A.3.56)}$$

$$R_{jc}^f = t_c^f (1-t_c^s) BQ_j \quad \text{(A.3.57)}$$

$$R_{jc}^s = t_c^s BQ_j \quad \text{(A.3.58)}$$

Noncorporate income tax base

$$BN_j = PKD_{jn} KD_{jn} - D_{jn} - IC_{nj} - R_{jn}^p \quad \text{(A.3.59)}$$

Individual capital income tax base

$$B_j^{IDV} = BQ_j - t_c BQ_j + BN_j + IC_{jc} + IC_{jn} \quad \text{(A.3.60)}$$

Individual capital income tax revenue from industry j; cash flow

$$t_c^{earn} = \left[\alpha^{DIV} t_c^e + \left(1 - \alpha^{DIV}\right) t_c^g \right] \quad \text{(A.3.61)}$$

$$R_j^{If} = \left[\alpha^{DIV} t_c^{ef} + \left(1 - \alpha^{DIV}\right) t_c^{gf} \right] \left(BQ_j - R_{jc}^f \right) + t_n^{ef} BN_j + t_n^{ef} \left(IC_{jc} + IC_{jn} \right) \quad \text{(A.3.62)}$$

$$R_j^{Is} = \left[\alpha^{DIV} t_c^{es} + \left(1 - \alpha^{DIV}\right) t_c^{gs} \right] \left(BQ_j - R_{jc}^s \right) + t_c^{es} BN_j + t_c^{es} \left(IC_{jc} + IC_{jn} \right) \quad \text{(A.3.63)}$$

$$CF_j = B_j^{IDV} - R_j^{If} - R_j^{Is} + DC_j + DN_j \quad \text{(A.3.64)}$$

$$= (1 - t_c)(PKD_{jc} KD_{jc} - D_{jc} - IC_{jc} - R_{jc}^p) + IC_{jc} + D_{jc} + PKD_{jn} KD_{jn} - R_{jn}^p - R_j^{If} - R_j^{Is}$$

Household property taxes

$$R_h^p = t_h^p PK_{t-1} \left(K_{hst-1} + K_{hlt-1} \right) \quad \text{(A.3.65)}$$

$$\text{debt}_h = \beta_h PK_{t-1} \left(K_{hst-1} + K_{hlt-1} \right) \quad \text{(A.3.66)}$$

$$BH = 0 - R_h^p - debt_h i_t \left(1 - \frac{PKD_{h,short} KD_{h,short}}{PKD_h KD_h}\right) \quad \text{(A.3.67)}$$

$$RKH^{eq} = \sum_j (R_j^{If} + R_j^{Is}) \quad \text{(A.3.68)}$$

$$RKH^{hh} = t_h^e BH \quad \text{(A.3.69)}$$

$$RKH^{int} = t_n^e (GINT^p + Y^{row}) \quad \text{(A.3.70)}$$

$$RKH = RKH^{eq} + RKH^{hh} + RKH^{int} \quad \text{(A.3.71)}$$

*Revenue summed over all industries: Property; Capital income; Wealth taxes

$$RP = \sum_j^{N_{IND}} R_{jc}^p + R_{jn}^p + R_h^p \quad \text{(A.3.72)}$$

$$RK^f = \sum_j^{N_{IND}} R_{jc}^f + R_j^{If} + BHt_h^{ef} + \frac{tk^f}{1 - tk} \left(rBG + rBF \right) + tk^f \left(GINT^p + Y^{row} \right) + \left(1 - tk^f\right) PKD_{35} KD_{35} \quad \text{(A.3.73)}$$

$$RK^s = \sum_j^{N_{IND}} R_{jc}^s + R_j^{Is} + BHt_h^{es} + \frac{tk^s}{1-tk}\left(rBG + rBF\right) + tk^s\left(GINT^p + Y^{row}\right) + \left(1-tk^s\right)PKD_{35}KD_{35} \quad \text{(A.3.74)}$$

$$RW^f = tw^f[\sum_{j=1}^{NIND}\sum_{s,l}\left(1-ITC_{cs}-t_c z_{cs}\right)PK_{t-1}K_{jcst-1} + \left(1-ITC_{ns}-t_n z_{ns}\right)PK_{t-1}K_{jnst-1} + BG_t + BF_t] \qquad f=f,s \quad \text{(A.3.75)}$$

$$Y^I = \sum_j^{N_{IND}} IC_{jc} + IC_{jn} + debt_h i_t \quad \text{(A.3.76)}$$

$$YK = \sum_j PKD_j KD_j \quad \text{(A.3.77)}$$

$$YK^{gov} = PKD_{35}KD_{35} - R_{35}^p - R_{35c} - R_{35}^I \quad \text{(A.3.78)}$$

$$YK^{bus} = YK - PKD_{35}KD_{35} - PKD_{36}KD_{36} \quad \text{(A.3.79)}$$

$$DEP^{tot} = \sum_j D_{jc} + D_{jn} \quad \text{(A.3.80)}$$

End of disaggregated capital version_____

A.4 The Government and pollution externalities

Tax rates

$$tc_i = tc + tc^g \qquad i \in I_{\text{COM}} \qquad \text{(A.4.1)}$$

$$tc_N = tc + tc^N \qquad \text{(A.4.2)}$$

$$tc_K = tc + tc^K \qquad \text{(A.4.3)}$$

$$tc_L = tc + tc^L \qquad \text{(A.4.4)}$$

$$tx_i^v = \sum_{x=1} tx_x^{Xv} XP_{ix} \qquad i \in I_{IND} \quad x \in I_{EXT} \qquad \text{(A.4.5)}$$

$$tx_i^u = \sum_{x=1} tx_x^{Xu} XP_{ix} \qquad \text{(A.4.6)}$$

$$tx_i^{rv} = \sum_{x=1} tx_x^{Xv} XM_{ix} \qquad \text{(A.4.7)}$$

$$tx_i^{ru} = \sum_{x=1} tx_x^{Xu} XM_{ix} \qquad \text{(A.4.8)}$$

$$tt_i^{\text{full}} = tt_i + tx_i^v + \frac{tu_i + tx_i^u}{PO_i} \qquad \text{(A.4.9)}$$

$$tl^0 = \sum_j PLD_j LD_j \left(1 - \frac{tl^a}{tl^m}\right) \qquad \text{(A.4.10)}$$

Stock-flow relations

$$BG_t = BG_{t-1} + \Delta G + GFI + \Delta P_t^{BGF} + BG^{disc} \qquad \text{(A.4.16)}$$

$$BG_t^* = BG_{t-1}^* - GFI - \Delta P_t^{BGF*} \qquad \text{(A.4.19)}$$

Revenues and expenditures

$$\begin{aligned} R\_TOTAL = {} & R\_SALES + R\_TARIFF + R\_P + R\_K + RK^{hh} + R\_L \\ & + R\_W + R^N + R\_UNIT + R\_EXT + R\_ITC \\ & + R\_CON - R\_CON^{reb} + R\_CON^{gov} + YK^{gov} + TLUMP \end{aligned} \qquad \text{(A.4.20)}$$

$$R\_SALES = \sum_j tt_j PO_j QI_j \qquad \text{(A.4.21)}$$

$$R\_TARIFF = \sum_i tr_i PM_i M_i \qquad \text{(A.4.22)}$$

$$R\_P = tpPK_{t-1} K_{t-1} \qquad \text{(A.4.23)}$$

$$R\_K = tk(\sum_{j=1,36} PKD_j KD_j - RK^{hh}) + \frac{tk}{1-tk} r(BG_{t-1} + BF_{t-1})$$
$$+ tkGINT^{adj} + tkY^{ROW,adj} \quad \text{(A.4.24)}$$
$$= tk(\sum_{j=1,36} PKD_j KD_j - RK^{hh}) + tkGINT + tkY^{ROW}$$

$$RK^{hh} = \frac{tk^{hh}}{1-tk^{hh}} PKD_{36} KD_{36} \quad \text{(A.4.25)}$$

$$R\_L = tl^a P^h LS/(1-tl^m) = tl^a \sum_j PLD_j LD_j \quad \text{(A.4.26)}$$

$$R\_W = tw(PK.K + BG + BF) \quad \text{(A.4.27)}$$

$$R\_UNIT = \sum_j tu_j QI_j \quad \text{(A.4.28)}$$

$$R\_EXT = \sum_j tx_j^v \left(PI_j QI_j + PM_j M_j\right) + \sum_j tx_j^u \left(QI_j + M_j\right) \quad \text{(A.4.29)}$$

$$R\_ITC = -t^{ITC} PII_t I_t^a \quad \text{(A.4.30)}$$

$$R\_CON^{marg} = \sum_{I_{COM}} tc_i PS_i C_i + RCON^{hk} \quad \text{(A.4.31)}$$

$$R\_CON^{hk} = (tc + tc^K)\frac{\psi_{36}^K KD_{36}}{KD} PII_t I_t^a \quad \text{(A.4.32)}$$

$$R\_CON^{reb} = tcVCC^{exempt} \quad \text{(A.4.33)}$$

$$R\_CON^{gov} = \frac{tc^G}{1+tc^G} VGG \quad \text{(A.4.34)}$$

$$R\_CON^{net} = R\_CON^{marg} - R\_CON^{reb} \quad \text{(A.4.35)}$$

R^N nontax receipts (C526)
GFI govt foreign net investment (C997)
$GINT^{row}$ govt net interest payments to foreigners (C921)
G^{tran} govt transfer payments to households (excl social insur) (C910)
$G^{tran,row}$ govt transfer payments to foreigners (C999)
$GINT^p$ govt interest payments to private bond holders (C920)
$GINT^{ss}$ investment income of social insur funds (C922)
R_SS transfers to govt from social insur funds for admin expenses (C914)
ΔG government deficit (C994)

$$EXP^{gengov} = VGG + G^{tran} + G^{tran,row} + G^{Ktran} + G^{Ktran,row} + GINT^p + GINT^{row} \quad \text{(A.4.36)}$$

$$G\_SS = GINT^{ss} - R\_SS \quad \text{(A.4.37)}$$

$$\Delta G = EXP^{gengov} - R\_TOTAL + G\_SS \quad \text{(A.4.38)}$$

$\Rightarrow$

$$VGG = \Delta G + R\_TOTAL + R\_SS$$
$$-GINT^p - GINT^{ss} - GINT^{row} - G^{TRAN} - G^{tran,row} \quad \text{(A.4.39)}$$

equivalently:

$$VGG = \Delta G + R\_TOTAL - r\frac{BG}{1-tk} - GINT^{adj} - GINT^{row} - G^{TRAN} - G^{tran.row} \quad \text{(A.4.40)}$$

$$GINT^{adj} = GINT^{p} + GINT^{ss} - R\_SS - \frac{r}{1-tk}BG_{t-1} \quad \text{(A.4.41)}$$

Alternatively, using "endogenous interest payments" option:

$$VGG = \Delta G + R\_TOTAL + R\_SS - r\frac{BG}{1-tk} - GINT^{row} - G^{TRAN} - G^{tran.row} \quad \text{(A.4.42)}$$

$$GINT^{adj} = 0 \quad \text{(A.4.42b)}$$

$$VG_i = \alpha_i^G VGG \qquad i \in I_{INP} \quad \text{(A.4.45)}$$

$$G_i = VG_i / PS_i \quad \text{(A.4.46)}$$

$$PLD_{Gt} LD_{Gt} = \alpha_L^G VGG_t \quad \text{(A.4.46b)}$$

$$PKD_{Gt} KD_{Gt} = \alpha_K^G VGG_t \quad \text{(A.4.46c)}$$

Vectors for use in A.6.2 below:

$$VG \equiv (PS_1 G_1, \ldots, PS_{35} G_{35})' \quad \text{(A.4.47)}$$

$$G^P \equiv (G_1, \ldots, G_{35})'$$

$$G \equiv (G_1, \ldots, G_{35}, NCI_G, KD_G, LD_G)'$$

$$PGG = \prod_i PS_i^{\alpha_i^G} \quad \text{(A.4.48)}$$

$$GG = VGG / PGG \quad \text{(A.4.49)}$$

Government closure options:

$$VGG_t = \begin{cases} R\_TOTAL + \Delta G + \ldots & 'resid' \\ \gamma_t^{VGG} GDP_t & 'propr' \end{cases} \quad \text{(A.4.50)}$$

$$g^{GDP} = \frac{VGG}{GDP} \quad \text{(A.4.51)}$$

Flat tax alternatives

AS=Armey-Shelby; HR=Hall-Rabushka; JCT=Joint Committee on Taxation

$$RL^z = \alpha_{FLAT}^f + \beta_{FLAT}^f \ln(YL^{gross}) \qquad \text{f=AS,HR} \quad \text{(A.4.54)}$$

$$= \alpha_{FLAT}^f + \beta_{FLAT}^f \ln(PCC * CC + VII + VGG) \qquad \text{f=JCT} \quad \text{(A.4.55)}$$

$$R\_FLAT^{hh} = tl^{flat}(YL^{gross} - RL^z) \quad \text{(A.4.56)}$$

$$\Rightarrow tl^{a} = \frac{R\_FLAT^{hh}}{YL^{gross}}; \qquad RL = R\_FLAT^{hh} \tag{A.4.57}$$

$$VII^{bus} = VII\,\frac{YK^{bus}}{YK} \tag{A.4.58}$$

$$BB^{flat} = YK^{bus} - VII^{bus}; \qquad \text{AS,HR} \tag{A.4.59}$$

$$= YK^{bus} - \sum_{j}(\delta_{cs}K_{jcst-1} + \delta_{cl}K_{jclt-1} + \delta_{ns}K_{jnst-1} + \delta_{nl}K_{jnlt-1}+)PK \quad \text{JCT} \tag{A.4.60}$$

$$R\_FLAT^{bus} = tl^{flat}BB^{flat} \tag{A.4.61}$$

$$R\_FLAT = R\_FLAT^{hh} + R\_FLAT^{bus} \tag{A.4.62}$$

___end of Flat tax alternatives

Externalities

$$EXT_{x} = \sum_{j} XP_{jx}QI_{j} + \sum_{j} XM_{ix}M_{i} \qquad x \in I_{EXT} \tag{A.4.63}$$

A.5 The Rest-of-the-World

Non-competitive imports

$$PNCI_j = e\left(1 + tr_j^n\right)PNCI_j^* \qquad j = 1,\ldots,35,C,I,G \qquad \text{(A.5.1)}$$

$$PNCI_i^{land} = ePNCI_j^* \qquad \text{(A.5.2)}$$

Competitive imports and domestic output making up total supply:

$$QS_i = QS\left(QC_i, M_i\right) \qquad i \in I_{COM} \qquad \text{(A.5.3)}$$

$$\begin{aligned}\ln PS_{it} &= \alpha_{ct}\ln PC_{it} + \alpha_{mt}\ln PM_{it} + \tfrac{1}{2}(\beta_{cc}\ln^2 PC_{it} + 2\beta_{cm}\ln PC_{it}\ln PM_{it} \\ &\quad + \beta_{mm}\ln^2 PM_{it}) + f_{ct}^M \ln PC_{it} + f_{mt}^M \ln PM_{it} \\ &\equiv a^M \ln P^{M_i} + \ln P^{M_i}{}' B^{M_i} \ln P^{M_i} + \ln P^{M_i}{}' f_t^M\end{aligned} \qquad \text{(A.5.4)}$$

$$\ln P^{M_i} \equiv \left(\ln PC_i, \ln PM_i\right)'$$

$$PM_i = e\left(1 + tr_i + tx_i^{rv}\right)PM_i^* + tx_i^{ru} \qquad i \in I_{COM} \qquad \text{(A.5.5)}$$

$$PM_i^{land} = ePM_i^* \qquad \text{(A.5.6)}$$

$$SD^i \equiv \begin{bmatrix} PC_iQC_i / PS_iQS_i \\ PM_iM_i / PS_iQS_i \end{bmatrix} = \alpha^{M_i} + B^{M_i}\ln P^{M_i} + f_t^{Mi} \qquad \text{(A.5.7)}$$

Cobb-Douglas option:

$$SD^i \equiv \begin{bmatrix} PC_iQC_i / PS_iQS_i \\ PM_iM_i / PS_iQS_i \end{bmatrix} = \alpha^{CD,M_i} \qquad \text{(A.5.7b)}$$

$$PS_iQS_i = PC_iQC_i + PM_iM_i \qquad i \in I_{COM} \qquad \text{(A.5.8)}$$

Vectors for use in A.6.5:

$$VQS \equiv \left(PS_1QS_1,\ldots,PS_{35}QS_{35}\right)' \qquad \text{(A.5.9)}$$

$$VM \equiv \left(PM_1M_1,\ldots,PM_{35}M_{35}\right)'$$

$$SM \equiv \left(SD_2^1, SD_2^2,\ldots,SD_2^{35}\right)'$$

$$M \equiv \left(M_1, M_2,\ldots,M_{35}\right)'$$

$$PS_i^C = (1 + tc_i)PS_i \qquad i \in I_{COM} \qquad \text{(A.5.10)}$$

$$\begin{aligned}PS_N^C &= (1 + tc_N)PNCI_C \\ PS_K^C &= (1 + tc_K)PKD_C \\ PS_L^C &= (1 + tc_L)PLD_C\end{aligned} \qquad \text{(A.5.11)}$$

$$\Delta \ln PM^{*}_{it} = \Delta f^{p}_{it} \quad t > 2005; \qquad PM^{*}_{it} = \text{data for } t = .., 2004, 2005 \tag{A.5.12}$$

Exports

Exports prior to version 16:

$$X_{it} = EX^{0}_{it}\left(\frac{\left(1+tr^{*}_{i}\right)PC_{it}}{e_t PM^{*}_{it}}\right)^{\eta_i} + X^{tr}_{it} \qquad i \in I_{COM} \tag{A.5.13}$$

$$EX^{0}_{it} = \alpha^{x}_{i} + \lambda^{x}_{i} \ln Y^{*}_{t} \tag{A.5.14}$$

Exports in version 16:

$$SX^{i} \equiv \frac{PS_i X^{IDEN}_{i}}{PS_i QS_i} = \alpha^{X_i} + B^{X_i} \ln P^{X_i} + f^{Xi}_{t} \tag{A.5.15}$$

$$\ln P^{X_i} \equiv \left(\ln e_t PM^{*}_{it}, \ln PC_{it}\right)'$$

$$X_{it} = X^{IDEN}_{it} + X^{tr}_{it} \tag{A.5.16}$$

$$X^{tr}_{i} = \frac{PCC.XR}{\sum PC_i C_i} C_i \tag{A.5.17}$$

Vectors used in A.6.2:

$$X \equiv \left(X_1, \ldots, X_{35}\right)' \tag{A.5.18}$$

$$VX \equiv \left(PC_1 X_1, \ldots, PC_{35} X_{35}\right)'$$

Current account and net foreign assets

$$V^{IMP} = \sum_i ePM^{*}_{i} M_i \tag{A.5.19}$$

$$V^{NCI} = \sum_j ePNCI^{*}_{j} NCI_j \tag{A.5.20}$$

$$V^{EX} = \sum_i PC_i X_i \tag{A.5.21}$$

$$TB = V^{EX} - V^{IMP} - V^{NCI} \tag{A.5.22}$$

$$TB^{*} = TB / e \tag{A.5.23}$$

$$CA = TB + Y^{row} - GINT^{row} - G^{tran,row} - G^{Ktran,row} - H^{row} \tag{A.5.24}$$

$$= TB + \frac{r}{1-tk} BF + Y^{row,adj} - GINT^{row} - G^{tran,row} - G^{Ktran,row} - H^{row} \tag{A.5.24b}$$

$$CA^{*} = CA / e \tag{A.5.25}$$

$$Y^{ROW,adj} = Y^{ROW} - \frac{r}{1-tk} BF \tag{A.5.26}$$

Stock-flow relation:

$$BF_t = BF_{t-1} + CA_t - GFI + BF^{\text{disc}} + \Delta P^{BF} \qquad \text{(A.5.27)}$$

CA current account surplus of the US (C986A)

Y^{row} net private factor income from rest-of-world (C454P)

A.6 Markets, Numeraire and National Accounting

Final demands

$$VFD_i = PS_i\left(C_i^P + I_i^P + G_i^P\right) + PC_i X_i \qquad i \in I_{COM} \tag{A.6.1}$$

$$\begin{aligned} VFD &\equiv (VFD_1, \ldots, VFD_{35})' \\ &= VC + VI + VG + VX \end{aligned} \tag{A.6.2}$$

Supply equal demand for commodities

$$\begin{aligned} PS_i QS_i &= \sum_{j=1}^{35} PS_i QP_i^j + VFD_i \\ &= \sum_{j=1}^{35} A_{ij} VQI_j + VFD_i \end{aligned} \qquad i \in I_{COM} \tag{A.6.3}$$

$$VQS = \mathbf{A}\ VQI + VFD \tag{A.6.4}$$

$$VQC = \text{Diag}(SM)VQS \qquad VQS = \text{Diag}(1/SM)VQC \tag{A.6.5}$$

$$\begin{aligned} &\text{Diag}(1/SM)VQC - \mathbf{A}\ VQI = VFD \\ &\text{Diag}(1/SM)\mathbf{M}'\text{Diag}(\iota + tt^{\text{full}})VQI - \mathbf{A}\ VQI = VFD \\ &\left[\text{Diag}(1/SM)\mathbf{M}'\text{Diag}(\iota + tt^{\text{full}}) - \mathbf{A}\right] VQI = VFD \end{aligned} \tag{A.6.6}$$

Saving-investment balance

$$\begin{aligned} VII &= S - \left(BG_t - BG_{t-1}\right) - \left(BF_t - BF_{t-1}\right) \\ &= S - \Delta G^{net} - CA \end{aligned} \tag{A.6.7}$$

Demand equal supply of capital

$$PKD_j = \psi_j^K PKD \qquad j \in I_{\text{BUY}} \tag{A.6.8}$$

$$\sum_{j=1}^{C} PKD_j KD_j = PKD.KD \tag{A.6.9}$$

$$\sum_{j=1}^{C} \psi_j^K KD_j = KD = \psi^K K_{t-1} \tag{A.6.10}$$

Demand equal supply of labor

$$PLD_j = \psi_j^L \frac{P^h}{\left(1-tl^m\right)} \qquad j \in I_{\text{BUY}} \qquad \text{(A.6.11)}$$

$$PN^R = \psi_C^R P^h \qquad \text{(A.6.12)}$$

$$YL^{gross} = VLD = \sum PLD_j LD_j \qquad \text{(A.6.13)}$$

$$\left(1-tl^m\right) YL^{gross} = P^h LS = P^h\left(\bar{L} - \psi_C^r N^R\right) \qquad \text{(A.6.14)}$$

$$\sum_{j=1}^{G} \psi_j^L LD_j = LS \qquad \text{(A.6.15)}$$

Disaggregated capital version ("version 9"): Arbitrage between different assets

$$i = \rho^{eq} - \pi^{eq} \qquad \text{(A.6.16)}$$

$$\rho^e = \bar{r}_0 PKD + \pi \qquad \text{(A.6.17)}$$

$\pi^{eq} = i(BAA) - \rho^e$ in USWS

$\bar{r}_0$ from model simulation trials

National Accounting

$$GDP = VCC + VII + VGG + V^{EX} - V^{IMP} - V^{NCI} \qquad \text{(A.6.18)}$$

$$GNP = GDP + Y^{ROW} - GINT^{row} - G^{tran,row} - H^{row} \qquad \text{(A.6.19)}$$

$$CC^{div} = divisia(C_i; PS_i^C) \qquad \text{(A.6.20)}$$

$$II^{div} = divisia(I_i; PS_i) \qquad \text{(A.6.21)}$$

$$GG^{div} = divisia(G_i; PS_i) \qquad \text{(A.6.22)}$$

$$X^{div} = divisia(X_i; PC_i) \qquad \text{(A.6.23)}$$

$$M^{div} = divisia(M_i, NCI_j) \qquad \text{(A.6.24)}$$

$$rGDP = divisia(CC^{div}, II^{div}, GG^{div}, X^{div}) - M^{div} \qquad \text{(A.6.25)}$$

Formula for $Q = divisia(q_i; p_i)$:

$$\ln\frac{Q_t}{Q_{t-1}} = \sum_{i=1}^{n} \tfrac{1}{2}\left(\frac{p_{it}q_{it}}{V_t} + \frac{p_{i,t-1}q_{i,t-1}}{V_{t-1}}\right) \ln\frac{q_{it}}{q_{i,t-1}}; \qquad V_t = \sum_{i=1}^{n} p_{it}q_{it}$$

Numeraire

$$P_t^h = \bar{P}_t^h \qquad \text{(A.6.30)}$$

Walras Law check

$$wal = [P^h\bar{L} - \left(1-tl^m\right)YL^{gross} - P^{leis}L^{leis}] / P^h\bar{L} \qquad \text{(A.6.31)}$$

A.7 Steady-state equilibrium

T denotes the terminal period that approximates the steady state.

$$\text{Prices}_T - \text{Prices}_{T-1} < tol \tag{A.7.1}$$

$$\text{Quantities}_T - \text{Quantities}_{T-1} < tol \tag{A.7.2}$$

$$\Delta G_T = 0 \tag{A.7.3}$$

$$CA_T = 0 \tag{A.7.4}$$

$$r_t = \rho \tag{A.7.5}$$

$$\psi^I I_T^a = \delta K^T \tag{A.7.6}$$

A.8 Glossary
8.1 Values; 8.1.2 Shares and Probabilities 8.2 Quantities; 8.3 Prices, interest; 8.4 Behaviorial Parameters; 8.5 Tax rates, Govt spending rates;

A.8.1 Values:

A		IO Use matrix; the use of commodities by each industry
A_j	$j \in I_{IND}$	Columns of **A**
A_{ij}	$j \in I_{COM}$ $j \in I_{IND}$	Share of input *i* in producing output *j*
BB^{flat}		Business tax base (flat tax)
BF		Net US private sector claims on rest-of-world
BF^{disc}		Stock-flow discrepancy in the US external accounts
BG		Government debt to domestic households
BG^{disc}		Stock-flow discrepancy in the US govt accounts
$BG*$		Government debt to rest-of-world
BN_j	$j \in I_{IND}$	Tax base of noncorp portion of capital income
BQ_j	$j \in I_{IND}$	Tax base of corp portion of capital income
CA		Current account surplus of the US
CF_j	$j \in I_{IND}$	Cash flow of industry j (version 9)
D_{jc}	$j \in I_{IND}$ $c \in I_{LEGAL}$	Depreciation deductions
$debt_h$	`	Debt financed portion of household capital
DEP^{tot}		Value of total depreciation
DIV		"Dividends"; after-tax capital income
EXP^{gengov}		Total expenditures of general govt
G^{Ktran}		Capital transfers from government to households
$G^{Ktran,row}$		Capital transfers from government to rest-of-world
G^{tran}		Government transfers to households
$G^{tran,row}$		Government transfers to rest-of-world
$G^{tran,SS}$		Transfers from gen gov to Social Ins Trust Fund
GDP		Value of Gross Domestic Product
GFI		Government net foreign investment
$GINT$		Government interest payments on public debt to households (including social insurance funds)
$GINT^{adj}$		Arbitrage adjustment for interest income on government bonds
$GINT^{p}$		Govt interest payments on public debt to persons (excluding social insurance funds)
$GINT^{row}$		Government interest payments to rest-of-world
$GINT^{ss}$		Govt interest payments on public debt to social insurance funds
GNP		Value of Gross National Product

GM		Government net imports
G_SS		Net gen govt payments to Soc Ins Trust Funds
H^{row}		Household transfers to rest-of-world
IC_{jc}	$j \in I_{IND}\ \ c \in I_{LEGAL}$	Interest cost of industry debt
$\mathbf{M}$		Input-output Make matrix
M_k		Expenditures by household *k*
MF^X		Full expenditures (incl. leisure), CEX basis
MF^N		Full expenditures (incl. leisure), NIPA basis
R_{jc}^f	$j \in I_{IND}$; $f \in I_{\mathrm{GOV}}$	Corporate income tax revenue from j; {federal, S&L}
R_j^{If}	$f \in I_{\mathrm{GOV}}$	Individual capital income tax revenue from industry *j*
R_{jc}^p	$j \in I_{IND}\ \ c \in I_{LEGAL}$	Property tax revenue from j; {corp, noncorp}
R_h^p		Property tax revenue from household
R^N		Non-tax receipts of the government
RHK		Revenue from indiv cap taxes; total
RK^{hh}		Revenue from household capital services tax
RKH^{eq}		Revenue from indiv cap taxes on equity
RKH^{hh}		Revenue from indiv cap taxes on HH capital
RKH^{int}		Revenue from indiv cap taxes on HH claims on government and ROW
R_CON^{gov}		Revenue from consumption taxes on government spending
R_CON^{hk}		Revenue from consumption taxes on household capital
R_CON^{marg}		Notional revenue from consumption taxes (ignoring the exemption/rebate)
R_CON^{net}		Revenue from consumption taxes
R_CON^{reb}		Rebate for consumption taxes
R_EXT		Revenue from externality taxes
R_FLAT		Revenue from flat taxes
R_FLAT^{bus}		Revenue from business flat tax
R_FLAT^{hh}		Revenue from household flat tax
R_ITC		Negative revenue from investment tax credit
R_K		Capital tax revenue
R_L		Revenue from labor income taxes
R_P		Revenue from property taxes
R_SALES		Revenue from sales taxes
R_SS		Transfers from Soc Ins for admin expenses
R_TARIFF		Revenue from tariffs on imports
R_TOTAL		Total tax revenue
R_UNIT		Revenue from new taxes on unit of output
R_W		Revenue from taxes on wealth (estate tax)
S		Private savings
$TLUMP$		Lump sum tax

TB		Trade balance
TB^*		Trade balance in foreign prices
V^{EX}		Total value of exports
V^{IMP}		Total value of competitive imports
V^{NCI}		Total value of non-comparable imports
V^{QC}		Vector of values of domestic commodity output
V^{QI}		Vector of values (to producer) of domestic industry output
VC		Vector of values of household purchases of commodities
VCC		Value of aggregate consumption (PCE)
VCC^{exempt}		Consumption tax exemption base
VFD		Vector of values of final demand for commodities
VG_i (VG)	$i \in I_{NCOM}$	Value of government demand for commodity i (vector)
VGG		Government spending on goods and services
VI		Vector of values of investment inputs
VII		Value of domestic private investment
VII^{bus}		Value of business investment (flat tax)
VII^{invy}		Value of inventory investment
VII^{fixed}		Value of fixed private investment
VK^{gain}		Value of aggregate capital gains
VK_j^{rep}	$j \in I_{IND}$	Replacement cost of capital stock
VN		Vector of values of household purchases of NIPA commodities
VP^j	$j \in I_{IND}$	Vector of values of input into industry j
VT^{QI}		Vector of values of domestic industry output inclusive of sales tax
VQS		Vector of values of total commodity supply
VX		Vector of values of commodity exports
W		Tangible wealth of private sector (households)
wal		percentage error in Walras Law check
WF		Full wealth of private sector (households)
XR		Travel exports: Expenditures by foreign tourists in U.S.
Y		Private Income
Y^I		Interest from debt portion of claims on all capital
Y^*		Exogenous projected rest-of-world income
Y^{row}		Net income from rest-of-world
$Y^{row,adj}$		Arbitrage adjustment for income from rest-of-world
YF		Full private income (including imputations on leisure)
YK		Capital income
YK^{bus}		Capital income from private business (flat tax)
YK^{gov}		Capital income from govt enterprises

YK^{net}		Private capital income after tax
YL		Labor income after tax
YL^{gross}		Value of labor income
ΔA_j^{TFP}	$j \in I_{IND}$	Total technical change in industry j
ΔG		Government deficit
ΔP^{BF}		Capital gains on net foreign assets
ΔP^{BG}		Capital gains on government bonds
ΔP^{BG*}		Capital gains on government liabilities to Row
ΔSC_i	$i \in I_{CNODE=top}$	Difference in cons shares, CEX vs NIPA basis

A.8.1.2 Shares and Probabilities

g^{GDP}		govt purchases share of GDP
m_{ji}^{col}	$i \in I_{COM}$	share of national commodity i made by industry j
m_{ji}^{row}	$j \in I_{IND}$	share of industry j's output going to commodity i
H		matrix converting NIPA PCE classification to IO commodity classification
s_i^{con}	$i \in I_{PCE}$	NIPA commodity shares of consumption
s_j^m	$j \in I_{IND}$; m=K,L,E,M	{capital, labor, energy, material} input cost shares for industry j
SC_i^N	$i \in I_{CNODE=top}$	Shares of household demand, top tier, NIPA basis
SC_i^X	$i \in I_{CNODE=top}$	Shares of household demand, top tier, CEX basis
SD^i	$i \in I_{COM}$	Shares of domestic output, imports in total supply of *i*
SF		Vector of shares of commodities and leisure in full consumption
SI^M	$m \in I_{INV}$	Shares of investment at node *m*
SM		Vector of shares of imports in total supply
SN^m	$m \in I_{CNODE}$	Shares of consumption at node *m*
SP^{jm}	$j \in I_{IND}$ $m \in I_{PNODE}$	Shares of production at node *m* of industry *i*
SX^i	$i \in I_{COM}$	Shares of total supply of *i* exported

A.8.2 Quantities

A^{agg}		Productivity shift term that applies to all industries
A_j^{TFP}	$j \in I_{IND}$	Productivity in industry j due to both exogenous and induced components; and shocks
C^P		Vector of quantities of consumption of produced commodities
C		Vector of consumption, commodities and non-produced goods

C_i	$i \in I_{INP}$	Consumption of IO commodity *i*
C_i^X	$i \in I_{CNODE=top}$	Consumption CEX basis, top node item i
CC		Aggregate real consumption (from simple Cobb-Douglas index)
CC^{div}		Divisia index of real Consumption
EX_{it}^0	$i \in I_{NCOM}$	Exogenously projected portion of export function
EXT_x	$x \in I_{EXT}$	Quantity of externality of type *x*
F		Full consumption (commodities and leisure)
G^P		Vector of government purchases of commodities
G		Vector of government purchases, commodities and non-produced goods
G_i	$i \in I_{NCOM}$	Government purchases of commodity *i*
GG		Real government final purchases (from CD index)
GG^{div}		Divisia index of real government final purchases
I^a		Aggregate investment in domestic capital stock
I		Vector of commodities used in aggregate investment
I^m	$m \in I_{INV}$	Investment aggregate *m*
I_i^f	$i \in I_{NCOM}$	Investment of commodity *i* in fixed investment
I_i	$i \in I_{NCOM}$	Investment of commodity *i* in domestic capital stock
II^{div}		Divisia index of real aggreg investment
K		Aggregate private domestic capital stock
$K_{4(oil)}$		Capital stock in "oil and gas mining"
KD		Quantity of aggregate capital input normalized such that its rental price is one
KD_j	$j \in I_{NBUY}$	Quantity of capital input into sector *j*
KD_{jcs}	$j \in I_{NBUY}$ $c = \{c, n, h\}$ $s \in I_{ASSET}$	Quantity of capital input into sector *j*, {corp, noncorp}, {short asset, long asset}
$\bar{L}$		Time endowment of economy
LD_j	$j \in I_{NBUY}$	Quantity of labor input into sector *j*
LS		Labor supply
M		Vector of competitive imports
M_i	$i \in I_{COM}$	Imports of (competitive) commodities
M^{div}		Divisia index of real Imports (compet and nci)
N^{eq}		Number of household equivalent members in economy
N^m	$m \in I_{CNODE}$	Consumption of NIPA aggregate *m*
N^R		Leisure quantity (NIPA units)
N_i	$i \in I_{PCE}$	Consumption of NIPA commodities
NCI_j	$j \in I_{NBUY}$	Non-competitive imports into sector *j*
QC_i	$i \in I_{COM}$	Total domestic output of commodity *i*
QI_j	$j \in I_{IND}$	Output industry *j*

QP^{jm}	$j \in I_{IND}$	$m \in I_{PNODE}$	Aggregate input *m* into industry *j*
QP_i^j	$i \in I_{COM}$	$j \in I_{IND}$	Input of commodity *i* into industry *j*
QS_i	$i \in I_{COM}$		Total supply of commodity *i*
$rGDP$			Divisia index of real GDP
X			Vector of exports
X_i	$i \in I_{COM}$		Exports of commodity *i*
X_i^{IDEN}	$i \in I_{COM}$		Exports that are explicitly identified in IO
X_i^{tr}			Travel exports of commodity *i*
X^{div}			Divisia index of real exports

***A.8.3. Prices**:*

e		"Exchange rate"
i^*		Interest rate on private U.S. owned foreign assets
i		Cost of capital return to debt
P^h		Price of total hours (work and leisure)
P^{Hm}	$m \in I_{CNODE}$	Vector of prices at node *m* of consumption function
P^{Im}	$m \in I_{INV}$	Vector of prices at node *m* of investment function
P^{Pjm}	$j \in I_{IND}$; $m \in I_{PNODE}$	Vector of prices at node *m* of industry *j*'s production function
PC_i	$i \in I_{COM}$	Price of domestically produced commodities
PC_i^X	$i \in I_{CNODE=top}$	Price of consumption CEX basis
PC_R^X		Price of leisure on CEX basis
PCC		Price of aggregate commodity consumption from simple Cobb-Douglas index
PF		Price of full consumption
PGG		Price of aggregate government consumption (Cobb-Douglas index)
PI_j	$j \in I_{IND}$	Price of industry output paid by buyers
PII		Price of aggregate investment goods
PII^m	$m \in I_{INV}$	Price of investment aggregate *m*
PII_{mi}	$mi \in I_{INVm}$	Union of above aggregate investment prices and supply prices
PK		Price of capital stock
PK^{gain}		Capital gain rate for aggregate capital
PKD		Rental price of aggregate capital
PKD_j	$j \in I_{BUY}$	Rental price of capital paid by producer
PKD_{oil}		Rental price of capital of "oil and gas mining"
PKD_{hs}	$s \in I_{ASSET}$	Rental price of household capital; short and long

PKD_{jc}	$c \in I_{LEGAL}$	Rental price of capital; corporate and noncorp
PKD_{jcs}	$s \in I_{ASSET}$	Rental price of capital; short and long-lived
PLD_j	$j \in I_{BUY}$	Price of labor paid by employers
PM_i	$i \in I_{COM}$	Price of competitive imports paid by importers
PM_i^*	$i \in I_{COM}$	World price of competitive imports
PM_i^{land}	$i \in I_{COM}$	Landed price of imports before tariffs
PN_n	$n \in I_{NIPA}$	Price of NIPA PCE commodity
PN^m	$m \in I_{CNODE}$	Price of consumption aggregate *m*
PN_{mi}	$mi \in I_{CNODEm}$	Union of above 2 sets of consumption prices
PN^R		Price of leisure (NIPA basis)
$PNCI_j^*$	$j \in I_{BUY}$	World price of non-competitive imports
$PNCI_j$	$j \in I_{BUY}$	Price of non-competitive imports paid by importers
$PNCI_j^{land}$	$j \in I_{BUY}$	Landed price of non-competitive imports before tariffs
PO_j	$j \in I_{IND}$	Price of industry output received by producer
PP^{jm}	$j \in I_{IND} \quad m \in I_{PNODE}$	Price of aggregate input *m* into industry *j*
PP_{mi}^j	$mi \in I_{PNODEm}$	Union of above set of aggregate production prices and prices of inputs
PS		Vector of supply prices
PS_i	$i \in I_{COM}$	Price of commodities to buyers
PS_i^C	$i \in I_{COM}$	Prices of commodities paid by the household sector (after consumption taxes)
r		After tax interest rate used in Euler equation
r_{jc}	$j \in I_{IND} \;\; c \in I_{LEGAL}$	Weighted (equity and debt) rate of return, corp and noncorp
r_h		Weighted (equity and debt) rate of return to household capital
r_{csj}^{net}	$j \in I_{IND}; c = \{c, n, h\}$ $s \in I_{ASSET}$	Net return on capital, {corp, noncorp, household}, {short, long}
r_c^{equ}	c={c,n,h}	Rate of return to equity; {corp, noncorp, household}
π		Inflation rate in cost of capital formula (version 9)
π^{eq}		Equity premium (over debt)
ρ^e		Cost of capital return to equity

A.8.4 Parameters of behavioral equations, Kalman filter terms:
Household functions

ρ		Pure rate of time preference
σ		Household intertemporal elasticity of substitution
α^{Hm}	$m \in I_{CNODE}$	Shares (at unit prices) of consumption at node *m*
B^{Hm}		Share elasticity of consumption (w.r.t. prices) at node *m*
B_{pA}		Coefficients on demographic characteristics of CC function
ξ^{dd}		Distribution coefficient in top tier household demand function
ξ^{L}		Coefficients of demographic terms in top tier household demand function
f_t^{Hm}	$m \in I_{CNODE}$	Latent variable for bias of consumption change, lower tiers
ψ_C^R		Aggregation constant of leisure
H		Bridge matrix linking NIPA "Personal Constant Expenditures" commodities to IO commodities

Production and commodity functions

α_0^j	$j \in I_{IND}$	Cost function constant
α^{Pjm}	$j \in I_{IND}$; $m \in I_{PNODE}$	Shares (at unit prices) of inputs into industry *j* at node *m*
B^{Pjm}	$j \in I_{IND}$	Share elasticity of input demands (w.r.t.) at node *m*
B_{pt}^j	$j \in I_{IND}$	Biases of technical change
f_t^{Pj}	$j \in I_{IND}$	Latent variable for bias of technical change, top tier
f_t^{j}	$j \in I_{IND}$	Latent variable for technical change, top tier
f_t^{Pjm}	$j \in I_{IND}$ $m \in I_{PNODE}$	Latent variable for bias of technical change, lower tiers
A^{agg}	$j \in I_{IND}$	Index of aggregate technology shock
ΔA^{agg}		Aggregate technology improvement
λ_j	$j \in I_{IND}$	Industry technology shock
M		IO Make matrix; the contribution of each industry to each commodity
$\boldsymbol{m}^{row}$		Row shares of Make matrix
$\boldsymbol{m}^{col}$		Column shares of Make matrix
δ		Depreciation rate (aggregate capital)
nfac_p	$m \in I_{PNODE}$	number of factors at node m
pfac_p	$m \in I_{PNODE}$	location in price vector of price of subaggregate m
lfac_p	$m \in I_{PNODE}$,5	location of component prices of node m

Capital input and cost of capital functions

α_{KD0}^{j}	$j \in I_{IND}$	Constant of industry capital input price function
α_{KD}^{j}	$j \in I_{IND}$	Shares (at unit prices) of inputs of industry capital input
B_{KD}^{j}	$j \in I_{IND}$	Share elasticity of components of industry capital input
α_{KD}^{jc}	$j \in I_{IND}$	Shares (at $p = 1$) of components of indus corporate cap input
B_{KD}^{jc}	$j \in I_{IND}$	Share elasticity of components of indus corporate cap input
α_{KD}^{jn}	$j \in I_{IND}$	Shares (at $p = 1$) of components of indus noncorporate cap input
B_{KD}^{jn}	$j \in I_{IND}$	Share elasticity of components of indus noncorporate cap input
α_{KD0}^{h}		Constant of household capital input price function
α_{KD}^{h}		Shares (at $p = 1$) of components of household capital input
B_{KD}^{h}		Shares of components of household capital input
δ		Depreciation rate (aggregate capital)
δ_{cs}	$c = c,n,h$	Rate of depreciation of short-lived capital stock
δ_{cl}	$c = c,n,h$	Rate of depreciation of long-lived capital stock
β_{jc}	$j \in I_{IND}$	Corporate debt-equity ratio, industry *j*
β_{jn}	$j \in I_{IND}$	Noncorporate debt-equity ratio, industry *j*
β_{h}		Debt-equity ratio, household
α^{DIV}		Dividend-payout ratio

Investment functions and capital stock functions

α^{IY}		Share of inventory investment in total investment
α_{i}^{IY}	$i \in I_{COM}$	Share of inventory investment going to commodity *i*
α^{Im}	$m \in I_{INV}$	Shares (at unit prices) of commodities at investment node *m*
B^{Im}	$m \in I_{INV}$	Shares elasticity of components of total investment at node *m*
λ^{I}		Shocks to investment cost function
ε^{I}		Shock to rate of capital formation
f_{t}^{Im}	$m \in I_{INODE}$	Latent variable for bias of investment change, lower tiers
ψ^{K}		Aggregation constant of capital services
ψ_{j}^{K}	$j \in I_{BUY}$	Aggregation constant of capital
ψ^{I}		Aggregation constant of investment goods
ψ^{PK}		Aggregation constant of price of capital stock

Trade functions

α^{Mi}	$i \in I_{COM}$	Shares (at unit prices) of domestic commodities and imports in total supply
B^{Mi}	$i \in I_{COM}$	Shares elasticity of components of total supply
f^{Mi}		Latent variable for bias of import change
η^{i}	$i \in I_{COM}$	Export price elasticities
α^{Xi}	$i \in I_{COM}$	Shares of exports in total supply
B^{Xi}	$i \in I_{COM}$	Shares elasticity of components of exports
f^{Xi}		Latent variable for bias of export change

Government functions

α_i^G	$i \in I_{INP}$	Share of government expenditures on *i*

Labor functions

ψ_j^L	$j \in I_{BUY}$	Aggregation constant of labor
ψ_C^R		Aggregation constant for aggregate leisure

Externalities functions

XP_{jx}	$j \in I_{IND}$	$x \in I_{EXT}$	Production externalities
XM_{ix}	$i \in I_{COM}$	$x \in I_{EXT}$	Import externalities

A.8.5 Tax rates, tax parameters, Govt and Funds spending rates:

α_{FLAT}^l	l = AS,HR,JCT	Flat tax base coefficient {Armey-Shelby, Hall-Rabushka, Joint Comm Taxn}
β_{FLAT}^l	l = AS,HR,JCT	Flat tax income coefficient {Armey-Shelby, Hall-Rabushka, Joint Comm Taxn}
γ_c^p	$c = c,n,h$	Deduction of property taxes (= 1 in version 9)
γ_c^j	$c = c,n,h$	Proportion of interest payments deducted before tax
γ_c^d		Proportion of dividends deducted before tax on corp.
γ_c^g	$c \in I_{LEGAL}$	Proportion of capital gains on corporate equities excluded from individual income for tax purposes
γ^{VGG}		parameter for setting govt purchases as share of GDP
dhi		Proportion of inflation premium in interest

		determined by indexing rule of household interest expense
t_c		Tax rate on corporate capital income (federal + S&L)
t_c^e	$c = c,n,h$	Tax on equity income (corporate, noncorporate, household)
t_c^{earn}		Average tax on personal corporate capital income
t_c^f	$f = f,s$	Statutory tax rate on corporate capital income; federal, S&L
t_c^g	$c = c,n,h$	Capital gains tax (corporate, noncorporate, household)
t_c^p	$c = c,n,h$	Property tax rate; {corporate, noncorp, household
t_{cs}^{ITC}	$c \in I_{LEGAL}\ s \in I_{ASSET}$	Rate of investment tax credit
t_h		Tax rate on household income used to adjust deductions
tc_i	$i \in I_{COM}$	Total tax rate on consumption commodity
tc		Consumption tax rate
tc^g		Consumption tax on goods only
tc^G		Consumption tax on govt spending
tc^K		Consumption tax on household capital input
tc^L		Consumption tax on private household labor
tc^N		Consumption tax on imports only (NCI)
tl^a		Average tax rate on labor income
tl^{af}	$f = f,s$	Average tax rate on labor income; federal, state
tl^{flat}		Flat tax rate on income
tl^m		Marginal tax rate on labor income
tl^{mf}	$f = f,s$	Marginal tax rate on labor income; federal, state
tl^0		Implied tax rate on labor income at zero income
tk		Tax rate on aggregate capital income (version 8)
tk^{hh}		Tax rate on household capital input (version 8)
tp		Tax rate on aggregate property (version 8)
tr_i	$i \in I_{COM}$	Tariff rate on competitive imports
tr_i^n	$i \in I_{BUY}$	Tariff rate on noncomp. imports
tr_i^*	$i \in I_{COM}$	World tariff rate on US exports
tt_j	$j \in I_{IND}$	Indirect business tax (sales tax)
tt_j^f	$f = f,s$	Indirect business tax; federal, state&local
tt_j^{full}	$j \in I_{IND}$	The full tax rate on sales

tu_i	$i \in I_{IND}$	Unit tax on quantities sold
tx_i^u	$i \in I_{IND}$	Total unit externalities tax on quantities sold
tx_i^v	$i \in I_{IND}$	Total externalities tax on sales
tx_i^{ru}	$i \in I_{COM}$	Total unit externalities tax on quantities imported
tx_i^{rv}	$i \in I_{COM}$	Total externalities tax on imports
tx_x^{Xu}	$x \in I_{EXT}$	Tax on one unit of externality x
tx_x^{Xv}	$x \in I_{EXT}$	Tax on one dollar of externality x
tw		Wealth tax rate (estate taxes)
tw^f	$f = f,s$	Wealth tax rate (estate taxes); {fed, S&L}
z_{cs}	$c \in I_{LEGAL}\ s \in I_{ASSET}$	Depreciation allowances for $1 of investment