Analyzing Environmental Policies with IGEM, an Intertemporal General Equilibrium Model of U.S. Growth and the Environment
Part 2

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Part 2. Analyzing Environmental Policies with IGEM

Chapter 1. Detailed Description of the Intertemporal General Equilibrium Model (IGEM) of the U.S. Economy.

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1. Detailed Description of the Intertemporal General Equilibrium Model of the U.S. Economy.

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In Chapter 1 of Part 1 we have presented a brief overview of the Intertemporal General Equilibrium Model (IGEM) of the U.S. Economy. In this chapter we describe the model in much greater detail. The defining characteristic of a general equilibrium model is that prices are determined together with quantities through the interactions between supply and demand. The production sector is central to the supply side of the model, while the household sector forms the core of the demand side. The model is completed by market-clearing conditions that determine supplies and demands for all commodities along with the corresponding prices.

Intertemporal equilibrium requires that market-clearing conditions are satisfied for each commodity at each point of time. In addition, the markets are linked by investments in capital goods and asset prices. Asset pricing imparts forward-looking dynamics to IGEM, since the price of an asset is equal the discounted value of the prices of capital services over the asset's future lifetime. Capital services are generated by stocks of assets accumulated through past investments, so that capital accumulation provides backward-looking dynamics for the model.

A distinguishing feature of IGEM is that parameters of the behavioral equations are estimated econometrically, rather than calibrated from estimates taken from the literature. We outline estimation of the demand side of the model in Chapter 2 and the supply side in Chapter 3. The demand side includes the household sector, investment demand, and demand for exports. The supply side includes production sub-models for 35 individual industries and the demand for imports.

In describing the model we use simplified notation to avoid a needless proliferation of symbols. A complete list of the 2000 equations of the model is given in Appendix A. Section A.8 of this Appendix provides a glossary of all the symbols used. We recall that Figure 1 in the overview chapter in Part 1 illustrates the flow of goods and factor services among the four main sectors of the economy – production, household, government, and rest of the world. The flows of payments among these sectors determine the expenditure patterns for the economy as a whole, including consumption, investment, government, exports, and imports.

1.1. Production

We focus attention on market-based policies, such as energy and environmental taxes and tradable permits. Environmental and energy taxes insert tax wedges between supply and demand prices and generate government revenue. The supply and demand for tradable permits can be modeled along with demands and supplies for commodities. The costs associated with market-based policies are determined through the price responses to changes in policy. The key to analyzing the economic impacts of energy and environmental policy is the substitutability among productive inputs, especially energy inputs, in response to price changes induced by policy.

While production patterns reflect substitutability among inputs in response to price changes, patterns of production also depend on changes in technology. In the long run the material well being of the population depends on the growth of productivity. In addition, the relative demands for inputs may be altered by biased technical change. For example, energy use may decline in intensity due to energy-saving changes in technology, as well as substitution away from higher-priced energy. A complete characterization of production requires both substitution and technical change.

Since the trends that underlie changes in the patterns of production are quite complex, the specification of models of production suitable for the analysis of energy and environmental policy requires econometric methods. Estimates are not available in the literature for the parameters that describe substitutability and technical change for all the industrial sectors that comprise the U.S. economy. An alternative approach would be to adopt assumed parameter values, such as elasticities of substitution equal to zero, as in input-output analysis. However, the vast scale of energy conservation since the energy crises of the 1970's has made the "fixed coefficients" assumption of input-output models totally implausible.

We subdivide the business sector into the 35 industries listed in Table 1.1.[1] The government and household sectors also used energy but are excluded from the business sector. Five of the industries are energy producers – Coal Mining (industry 3), Oil and Gas Mining (4), Petroleum Refining (16), Electric Utilities (30), and Gas Utilities (31). We have chosen the classification of the non-energy industries to distinguish among sectors that differ in the intensity of utilization of different inputs, especially energy, and exhibit different patterns of technical change.

The output of the production sector is divided among 35 commodities, each the primary product of one of the 35 industries. For example, steel is a primary product of the Primary Metals industry, while brokerage services are included in the Finance, Insurance, and Real Estate industry. Many industries produce secondary products as well, for example, Petroleum Refining produces commodities that are the primary outputs of the Chemicals industry. The model permits joint production of this kind for all sectors and all commodities. We model joint production along with substitution and technical change for each industry.

The parameters of our production model are estimated econometrically from a historical data base covering the period 1960-2005. The data base is described in Appendices B, C, and D, and includes a time series of input-output tables in current and constant prices, as well as data on the prices and quantities of capital and labor services.[2] These data comprise the industry-level production account of the "new architecture" for the U.S. national accounts developed by Jorgenson (2009) and Jorgenson and Landefeld (2006, 2009).

The input-output tables consist of *use* and *make* matrices for each year. The use matrix gives the inputs used by each industry – intermediate inputs supplied by other industries, noncompeting imports, capital services and labor services. This matrix also gives commodity use by each category of final demand – consumption, investment, government, exports, and imports. The use matrix is illustrated in Figure 1.1. The rows of this table correspond to commodities, while the columns correspond to industries. Each entry in the table is the amount of a given commodity used by a particular industry. The sum of all entries in a row is equal to the total demand for the commodity, while the sum of all entries in a column is the value of all the inputs used in a given industry.

The make matrix describes an essential part of the technology of the U.S. economy. The rows of the make matrix correspond to industries, while the columns

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[1] See also Table 1.1 in Part 1.
[2] The methodology and data sources are presented in much greater detail by Jorgenson, Ho, and Stiroh (2005).

correspond to commodities. Each entry in the table is the amount of a commodity supplied or "made" by a given industry. The domestic supply of the commodity is the sum of all the entries in the corresponding column, while the sum of all the entries in a row represents the value of the commodities produced by a given industry. In short, the inter-industry accounts of the system of U.S. national accounts are critical components of the historical data base for IGEM. The inter-industry accounts in current and constant prices are discussed in detail in Appendix B and Chapter 4 of Jorgenson, Ho, and Stiroh (2005).

The use matrix includes values of capital and labor inputs. Like the inter-industry flows, these values are divided into prices and quantities. The value of capital services consists of all property-type income – profits and other operating surplus, depreciation, and taxes on property and property-type income. The price of capital services consists of the price of the corresponding asset, multiplied by an annualization factor that we denote the *cost of capital*. The cost of capital consists of the rate of return, the rate of depreciation, less capital gains or plus capital losses, all adjusted for taxes. The price of capital services and the cost of capital are discussed in much greater detail in Appendix D, in Jorgenson and Yun (2001), and in Chapter 5 of Jorgenson, Ho, and Stiroh (2005).

The quantity of capital services is the annual flow from a given type of asset. The assets included in our historical data base are plant, equipment, inventories, and land. Plant and equipment are sub-divided into detailed categories. For example, equipment includes different types of equipment, ranging from motor vehicles and construction equipment to computers and software. We construct prices and quantities of capital services for each industrial sector by aggregating prices and quantities of capital services to obtain a price and a quantity index of capital services for each sector. Each type of capital stock is weighted by the rental price of capital services, rather than the asset price used in aggregating different types of assets to obtain capital stocks.

Similarly, the value of labor services includes all labor-type income – wages and salaries, labor income from self-employment, supplements such as contributions to social insurance, and taxes such as payroll taxes. The quantity of labor input for each demographic category of labor input is hours worked. We construct a price and a quantity index of labor services by aggregating over the prices and quantities of labor services from the different demographic groups employed in each sector. These range from young workers with only secondary education to mature workers with advanced degrees. The price and quantity of labor services are described in more detail in Appendix C and Chapter 6 of Jorgenson, Ho, and Stiroh (2005).

Our methods for aggregating over detailed categories of capital and labor services are an essential feature of our historical data set. Arithmetic aggregates consisting of simple sums of hours worked or asset quantities would not capture the substitution possibilities within each aggregate. For example, a simple sum of computers and software with industrial buildings would fail to reflect the impact on capital input of a shift in composition toward information technology equipment and software. Similarly, a simple sum of hours worked over college-educated and non-college workers would not reveal the impact on labor input of increases in educational attainment of the U.S. work force.

1.1.1 Notation

To describe our submodel for producer behavior we begin with some notation. The general system is to use P for prices and Greek letters for parameters.

QI_j	quantity of output of industry *j*
PO_j	price of output to producers in industry *j*
PI_j	price of output to purchasers from industry *j*
$QP_{i}^{j}{}_{j}$	quantity of commodity input *i* into industry *j*
PS_i	price of commodity *i* to buyers
KD_j	quantity of capital input into *j*
LD_j	quantity of labor input into *j*
E_j	index of energy intermediate input into *j*
M_j	index of total nonenergy intermediate input into *j*
$P_{E,j}$	price of energy intermediate input into *j*
$P_{M,j}$	price of total nonenergy intermediate input into *j*
PKD_j	price of total capital input to industry *j*
PLD_j	price of total labor input to industry *j*
v	value shares
QC_i	quantity of domestically produced commodity *i*
PC_i	price of domestically produced commodity *i*
$M_{j,i}$	MAKE matrix; value of commodity *i* made by industry *j*

1.1.2 Top tier production function with technical change

The production function represents output from capital services, labor services, and intermediate inputs. Output also depends on the level of technology *t*, so that for industry *j*:

(1.1) $$QI_j = f(KD_j, LD_j, QP_1^j, QP_2^j, ...QP_m^j, t),\qquad (j=1,2,...,35)$$

This form of the production function is intractable as it stands. We assume that the production function is separable in energy and materials inputs, so that output at the first stage of the production model depends on quantities of energy input and input of non-energy materials, as well as inputs of capital and labor services:

(1.2) $$\begin{aligned} QI_j &= f(KD_j, LD_j, E_j, M_j, t); \\ E_j &= E(QP_3^j ...); \qquad M_j = M(QP_1^j, ...) \end{aligned}$$

In the second stage of the production model the energy and non-energy inputs depend on the components of each of the aggregates. For example, energy input depends on inputs of coal, crude oil, refined petroleum products, natural gas, and electricity. These are primary outputs of Coal Mining, Oil and Gas Mining, Petroleum Refining, Electric Utilities, and Gas Utilities. Similarly, non-energy input depends on all the non-energy commodities listed in Table 1.1. This includes materials such as primary and fabricated metals and services such as financial services.

We assume constant returns to scale and competitive markets, so that the production function (1.1) is homogeneous of degree one and the value of output is equal to the sum of the values of all inputs:

(1.3) $$\begin{aligned} PO_{jt}QI_{jt} &= PKD_{jt}KD_{jt} + PLD_{jt}LD_{jt} + P_{Ejt}E_{jt} + P_{Mjt}M_{jt} \\ P_{Ejt}E_{jt} &= PS_{3t}QP_{3t}^j + PS_{4t}QP_{4t}^j + .. + PS_{31t}QP_{31t}^j \\ P_{Mjt}M_{jt} &= PS_{1t}QP_{1t}^j + PS_{2t}QP_{2t}^j + .. + PS_{NCI,t}QP_{NCI,t}^j \end{aligned}$$

In order to characterize substitution and technical change we find it more convenient to work with the price function, rather than the production function (1.1)[3].

[3] The price function contains the same information about technology as the production function. For further detail, see Jorgenson (1986).

The price function expresses the price of output as a function of the input prices and technology, so that for industry j:

$PO_j = p(PKD_j, PLD_j, P_{Ej}, P_{Mj}, t)$.

We have chosen the translog form of the price function, so that substitutability can be characterized in a flexible manner and changes in technology can be represented by latent variables through the Kalman filter, as in Jin and Jorgenson (2009):

$$(1.4) \qquad \ln PO_t = \alpha_0 + \sum_i \alpha_i \ln p_{it} + \tfrac{1}{2}\sum_{i,k} \beta_{ik} \ln p_{it} \ln p_{kt} + \sum_i \ln p_{it} f_{it}^p + f_t^p$$

$$p_i, p_k = \{PKD, PLD, P_E, P_M\}$$

α_i, β_{ik} and α_0 are parameters that are separately estimated for each industry, and we have dropped the industry *j* subscript for simplicity.

The vector of latent variables

$$\xi_t = (1, f_{Kt}^p, f_{Lt}^p, f_{Et}^p, f_{Mt}^p, \Delta f_t^p)'$$

is generated by a first-order vector autoregressive scheme:

$$(1.5) \qquad \xi_t = F\xi_{t-1} + v_t .$$

The production sub-model (1.4) and (1.5) achieves considerable flexibility in the representation of substitution and technical change. An important advantage of this model is that it generates equations for the input shares that are linear in the logarithms of the prices and the latent variables. Differentiating equation (1.4) with respect to the logarithms of the prices, we obtain equations for the shares of inputs. For example, if we differentiate with respect to the price of capital services, we obtain the share of capital input:

$$(1.6) \qquad v_K = \frac{PKD_t KD_t}{PO_t QI_t} = \alpha_K + \sum_k \beta_{Kk} \ln P_k + f_{Kt}^p .$$

The parameters $\{\beta_{ik}\}$ are *share elasticities*, giving the change in the share of the *i*th input in the value of output with respect to a proportional change in the price of the *k*th input. These parameters represent the degree of substitutability among the capital (K), labor (L), energy (E), and non-energy (M) inputs. If the share elasticity is positive, the value share increases with a change in the price of the input, while if the share elasticity is negative, the share decreases with a change in the price. A share elasticity equal to zero implies that the value share is constant, as in a linear-logarithmic or Cobb-Douglas specification of the technology.

The price function is homogeneous of degree one, so that a doubling of input prices results in a doubling of the output price. This implies that the row and column sums of the matrix of share elasticities must be equal to zero:

(1.7) $\sum_i \beta_{ik} = 0$ for each k; $\sum_k \beta_{ik} = 0$ for each i.

Symmetry of the price effects implies that the matrix of share elasticities is symmetric. Monotonicity and concavity of the price function are discussed in Chapter 3.

The level of technology f_t^p, together with the biases of technical change $\{ f_{it}^p \}$, evolve according to equation (1.5). The latent variable f_t^p represents the level of technology. The first difference of the level of technology takes the form:

(1.8) $\Delta f_t^p = F_{p1} + F_{pK} f_{K,t-1}^p + F_{pL} f_{L,t-1}^p + F_{pE} f_{E,t-1}^p + F_{pM} f_{M,t-1}^p + F_{pp} \Delta f_{t-1}^p + v_{pt}$

A more detailed description of the production sub-model, including the price function, is presented in Chapter 3. Here we describe a few of the key features of this model.

The latent variables $\{ f_{it}^p \}$ describe the *biases of technical change*. A decrease in one of these latent variables implies that the input share decreases as technology changes. For example, if the energy share declines, holding prices of all inputs constant, the bias with respect to energy is negative and we say that technical change is energy-saving. Similarly, a positive bias implies that technical change is energy-using. Note that while the parameters describing substitution are constant, reflecting responses to varying price changes, the biases of technical change are may vary from time to time, since historical patterns involve both energy-using and energy-saving technical change.

1.1.3 Lower tier production functions for intermediate inputs

In modeling producer behavior in Section 1.1.2 we have introduced multi-stage allocation in order to avoid an intractable specification of the production function (1.1). In the lower tiers of the model energy and non-energy materials inputs are allocated to the individual commodities. Repeating the second stage from (1.2):

(1.9) $E_j = E(QP_3^j, QP_4^j, QP_{16}^j, QP_{30}^j, QP_{31}^j)$; $M_j = M(QP_1^j, ... QP_{NCI}^j)$.

To illustrate the elements of the tier structure we consider the price function for energy input:

$$(1.10)\quad \ln P_{Et} = \alpha_0 + \sum_{i \in energy} \alpha_i \ln P_{it}^{P,E} + \tfrac{1}{2}\sum_{i,k} \beta_{ik} \ln P_{it}^{P,E} \ln P_{kt}^{P,E} + \sum_{i \in energy} f_{it}^{node=E} \ln P_{it}^{P,E}$$

$$P_i^{P,E} \in \{PS_3, PS_4, PS_{16}, PS_{30}, PS_{31}\}$$

while the share equations are:

$$(1.11)\quad v_3 = \frac{PS_3 QP_3}{P_E E} = \alpha_3 + \sum_{k \in energy} \beta_{3k} \ln P_k^{P,E} + f_{3t}^{node=E}\,,$$

for coal mining, the first component, and so on, for components corresponding to crude petroleum, refined petroleum products, electricity, and natural gas.

The components of the non-energy materials (M) input include the other thirty commodities in Table 1.1, in addition to noncompeting imports, a commodity not produced by any domestic industry. This is denoted X_{NCI} in Figure 1.1. We model the demand for individual commodities within the non-energy materials aggregate for each industry j by means of a hierarchical tier structure of translog price functions.

The price functions for the sub-tiers (1.10) differ from the price function (1.4), since there is no latent variable representing the level of technology. This reflects the fact that the price of energy is an index number constructed from the prices of the individual components, while the price of output is measured separately from the prices of capital, labor, energy, and non-energy materials inputs. The price of output could fall, relative to the input prices, as productivity rises. As before, the parameters $\{\beta_{ik}\}$ are share elasticities representing the degree of substitutability among the five types of energy.

The latent variables of the Kalman filter $\{f_{it}^{node}\}$ represent the biases of technical change. For example, an increase in the latent variable $f_{30t}^{node=E}$ implies that the electricity share of total energy input is increasing, so that technical change is electricity-using, while a decrease in this latent variable implies that technical changes in electricity-saving. The latent variables are generated by a vector autoregression, as in (1.5).

The long list of commodities included among the non-energy materials inputs requires that the sub-models of these inputs must be arranged in a hierarchical fashion. The tier structure for producer behavior in each industry is given in Table 1.2. The non-energy materials input consists of five sub-aggregates – construction, agriculture materials, metal materials, non-metal materials, services. Each of these sub-aggregates in turn is a function of sub-groups, and so on, until all the *31* commodities are included. Each node to the tier structure employs a price function like equation (1.10).

1.1.4 Relation between commodities and industries, and output taxes.

Production or sales taxes are proportional to the price of output. These taxes are included for all 35 sectors of the production model and introduce wedges between the prices faced by sellers and buyers of the corresponding commodities. We have noted above that each industry makes a primary commodity and many industries make secondary products that are the primary outputs of other industries.

Denoting the buyer's price of the output of industry *j* by PI_j, we have:

(1.12) $PI_j = (1 + tt_j^{full})PO_j$,

The value of industry *j's* output is $VT_j^{QI} = PI_j QI_j$. We denote the price, quantity, and value of commodity *i* by, PC_i, QC_i and V_i^{QC} respectively, all from the purchasers' point of view. For column *i*, let the shares contributed by the various industries to that commodity in the base year be denoted:

(1.13) $m_{ji} = \dfrac{M_{ji,t=T}}{V_{i,t=T}^{QC}}$; $\sum_j m_{ji} = 1$

For row *j*, let the shares of the output of industry *j* be allocated to the various commodities be denoted:

(1.14) $m_{ji}^{row} = \dfrac{M_{ji,t=T}}{VT_{j,t=T}^{QI}}$; $\sum_i m_{ji}^{row} = 1$

The shares (1.13) and (1.14) are fixed for all periods after the base year. We assume that the production function for each commodity is a linear logarithmic or Cobb-Douglas aggregate of the outputs from the various industries. The weights are the base year shares. That is, we write the price of commodity *i* as:

(1.15) $PC_i = PI_1^{m_{1i}} ... PI_m^{m_{mi}}$ for i=1,2,....35

The values and quantities are given by:

(1.16) $V_{it}^{QC} = \sum_j m_{ji}^{row} PI_{jt} QI_{jt}$ for i=1,2,....35

(1.17) . $QC_i = \frac{V_i^{QC}}{PC_i}$

1.2. Household model

Policies that affect energy prices have different impacts on different households. On average households in warmer regions have larger electricity bills for cooling, elderly persons drive less, and households with children drive more. To capture these differences we subdivide the household sector into demographic groups. We treat each household as a consuming unit, that is, a unit with preferences over commodities and leisure.

Our household model has three stages. At the first stage lifetime income is allocated between consumption and savings. Consumption consists of commodities and leisure and we refer to this as *full consumption*. In the second stage full consumption is allocated to leisure and three commodity groups – nondurables, capital services, and services. In the third stage the three commodity groups are allocated to the 36 commodities, including the five types of energy. We now describe the three stages of the model, beginning with a definition of the symbols used.

1.2.1 Notation

A_k	vector of demographic characteristics of household k
C_i^X	quantity of consumption of commodity i
C_{ik}	quantity of consumption of commodity i by household k
F_t	quantity of full consumption
R_t	quantity of aggregate leisure
R_k^m	quantity of leisure
LS_t	quantity of aggregate labor supply
m_k	value of full expenditures of household k
KS_t	quantity of aggregate capital stock at end of period t
n_t	growth rate of population
PF_t	price of F_t
P_t^L	price of labor to employer, economy average

P_t^K	rental price of capital, economy average
r_t	rate of return between t-1 and t
Y_t	household disposable income
S_t	household savings

1.2.2 Household model 1st stage, intertemporal optimization.

Let V_{kt} denote the utility of household *k* derived from consuming goods and leisure during period *t*. In the first stage household *k* maximizes an additively separable intertemporal utility function:

$$(1.18)\quad \max_{F_{kt}} U_k = E_t\{\sum_{t=1}^{T}(1+\delta)^{-(t-1)}[\frac{V_{kt}^{(1-\sigma)}}{(1-\sigma)}]\}$$

subject to the lifetime budget constraint:

$$(1.19)\quad \sum_{t=1}^{T}(1+r_t)^{-(t-1)}PF_{kt}F_{kt} \le W_k$$

where F_{kt} is the full consumption in period *t*, F_{kt} is its price, r_t is the nominal interest rate, and W_k is the "full wealth" at time 0. σ is an inter-temporal curvature parameter, and δ is the subjective rate of time preference. The within-period utility function is logarithmic if σ is equal to one:

$$(1.18b)\ \max_{F_{kt}} U_k = E_t\{\sum_{t=1}^{T}(1+\delta)^{-(t-1)}\ln V_{kt}\}\ .$$

The term full wealth refers to the present value of future earnings from the supply of tangible assets and labor, plus transfers from the government and imputations for the value of leisure. Tangible assets include domestic capital, government bonds and net foreign assets.

Equations (1.18) and (1.19) are standard in growth models found in macroeconomics textbooks[4] and we note here that the optimality condition is expressed in an Euler equation:

$$(1.20)\quad \Delta\ln PF_{k,t+1}F_{k,t+1} = (1-\sigma)\Delta\ln V_{k,t+1} + \Delta\ln(-D(p_{k,t+1})) + \ln(1+r_{t+1}) - \ln(1+\delta)$$

where $D(p_{kt})$ is a function of the prices of goods and leisure given in (1.29) below.

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[4] See, for example, Barro and Sala-i-Martin (2003).

Chapter 2 describes how this household Euler equation is estimated using synthetic cohorts by adding over all households in each cohort. From this we derive an aggregate Euler equation. This Euler equation is forward-looking, so that the current level of full consumption incorporates expectations about all future prices and discount rates. The future prices and discount rates determined by the model enter full consumption for earlier periods through the assumption of rational expectations. The solution of the model includes forward-looking dynamics in every period. From the value of full consumption in any period we have the key elements to derive the savings in that period.

The above structure describes the most detailed implementation of the intertemporal stage in IGEM. In version 16 of IGEM we use a simpler version with an aggregate Euler equation derived from aggregate goods consumption and aggregate leisure. As described in Chapter 2, this aggregate Euler equation (2.30) is simply:

$$(1.20b)\quad \frac{F_t}{F_{t-1}} = \frac{(1+n_t)(1+r_t)}{1+\rho}\frac{PF_{t-1}}{PF_t}$$

where n_t is the rate of growth of population.

1.2.3 Household model 2nd stage, goods and leisure.

In the second stage of the household model full consumption is divided between the value of leisure time and personal consumption expenditures on commodities. Given the time endowment of the household sector, the choice of leisure time also determines the supply of labor. The allocation of full consumption employs a very detailed household demand model that incorporates demographic characteristics of the population. The data base for this model includes the Consumer Expenditure Survey (CEX) and Personal Consumption Expenditures (PCE) from the U.S. National Income and Product Accounts.

Conceptually, we determine the consumption C_{ik}^X of commodity *i* for household *k* by maximizing a utility function $U(C_{1k}^X,..C_{ik}^X..,C_{Rk}^X;A_k)$, where C_{Rk}^X is leisure and A_k denotes the demographic characteristics of household *k,* such as the number of children and age of head of household. Summation over all households gives the total demand for commodity *i*:

(1.21) $PC_{it}^{X}C_{it}^{X} = \sum_{k} P_{ikt}^{CX}C_{ikt}^{X}$ i=1,2,…,R

The price P_{ik}^{CX} is the price of good *i* faced by household *k*, the superscript X denotes that this is a CEX measure that must be distinguished from measures based on the National Income and Product Accounts and Inter-industry Transactions Accounts discussed below. Similarly, total leisure demand is the sum over all household's leisure demands ($\sum_{k} P_{Rkt}^{CX}C_{Rkt}^{X}$), and the sum of goods and leisure gives the full consumption determined is stage 1:

(1.22) $PF_tF_t = \sum_{i} PC_{it}^{X}C_{it}^{X} + PC_RC_R^{X}$

The list of commodities included in the household model is presented in Table 1.3 along with the values in 2005. These are defined in terms of categories of Personal Consumption Expenditures (PCE) and the last column of the table gives the precise definitions. One major difference between our classification system and the PCE is the treatment of consumers' durables. Purchases of new housing are included in investment in the NIPAs, while only the annual rental value of housing is included in the PCE.

Purchases of consumer durables such as automobiles are treated as consumption expenditures in the PCE. In the new architecture for the U.S. national accounts discussed by Jorgenson (2009), purchases are included in investment and annual rental values are treated as consumption. This has the advantage of achieving symmetry in the treatment of housing and consumers' durables. The annual flow of capital services from these assets is given as item 35 in Table 1.3.

A utility function written as $U(C_{1k}^{X},...,C_{Rk}^{X};A_k)$ is intractable. Accordingly, we impose a tier structure much like the demand from intermediate inputs in the production model of Section 1.1. At the top tier utility function depends on nondurables, capital services, services, and leisure:

(1.23) $U = U(C_{ND,k}, C_{K,k}, C_{SV,k}, C_{R,k}; A_k)$

$C_{ND} = C(C_1, C_2, ...C_{16})$; $C_{SV} = C(C_{17}, ...C_{NCI})$

Consumer nondurables (C_{ND}) and services (C_{SV}) are further allocated to the 36 commodities in the third stage of the household model. For the remainder on this subsection we focus on the top tier. We first describe the CEX data used to estimate the parameters of the household model. We then indicate how the model for individual households is aggregated to obtain the model of the household sector in IGEM.

In order to characterize substitutability among leisure and the commodity groups, we find it convenient to derive household *k*'s demands from a translog indirect utility function $V(p_k, m_k; A_k)$, where:

$$(1.24)\quad -\ln V_k = \alpha_0 + \alpha^H \ln\frac{p_k}{m_k} + \tfrac{1}{2}\ln\frac{p_k}{m_k}' B^H \ln\frac{p_k}{m_k} + \ln\frac{p_k}{m_k}' B_A A_k\,.$$

where p_k is a vector of prices faced by household k, α^H is a vector of parameters, B^H and B_A are matrices of parameters that describe price, total expenditure, and demographic effects and A_k is a vector of variables that describe the demographic characteristics of household *k*.[5] The value of full expenditure on leisure and the three commodity groups is:

$$(1.25)\quad m_k = P^C_{ND} C_{NDk} + P^C_K C_{Kk} + P^C_{SV} C_{SVk} + P^C_R C_{Rk}\,.$$

In (1.24) demands are allowed to be non-homothetic, so that full expenditure elasticities are not required to be equal to unity.

The commodity groups in (1.23) and (1.24) represent consumption of these commodities by household *k*. The leisure consumed by household *k* is a more complicated measure, since we have to take into account the different opportunity costs of time of different members of the household. We assume that the effective quantity of leisure of person *m* (R^m_k) is non-work hours multiplied by the after tax wage, relative to the base wage $q^m_k = p^m_R / p^0_R$.

We assume a time endowment of $\bar{H} = 14$ hours a day for each adult. The annual leisure of person *m* is the time endowment less hours worked *LS*:

$$(1.26)\quad R^m_k = q^m_k(\bar{H}^m_k - LS^m_k) = q^m_k(14*365 - \text{hours worked}^m_k)$$

The quantity of leisure for household *k* is the sum over all adult members:

$$(1.27)\quad C_{Rk} = \sum_m R^m_k$$

and the value is:

[5] The aggregation properties of this indirect utility function is discussed in Jorgenson and Slesnick (2008).

(1.28) $P_R^C C_{Rk} = p_R^0 \sum_m R_k^m = \sum_m p_R^m (\bar{H}_k^m - LS_k^m)$

The demand functions for commodities and leisure are derived from the indirect utility function (1.24) by applying Roy's Identity:

(1.29) $\mathbf{w}_k = \frac{1}{D(p_k)}(\alpha^H + B^H \ln p_k - \iota' B^H \ln m_k + B_A A_k)$

where $\mathbf{w}_k$ is the vector of shares of full consumption, ι is a vector of ones, and $D(p_k) = -1 + \iota' B^H \ln p_k$. For example, the demand for consumer nondurables is::

(1.30) $w_{ND,k} = \frac{1}{D(p_k)}(\alpha_{ND}^H + B_{ND\bullet}^H \ln p_k - \iota B^H \ln m_k + B_{A,ND\bullet} A_k)$

where $B_{ND\bullet}^H$ denotes the top row of the B^H matrix of share elasticities.

We require that the indirect utility function must obey the restrictions:

(1.31) $B^H = B^{H}{}'; \quad \iota' B^H \iota = 0, \quad \iota' B_A = 0, \quad \iota' \alpha^H = -1,$

where B^H are the share elasticities, $\iota' B^H$ represents the full expenditure effect, and the k[th] column of B_A determines how the demands of demographic group *k* differs from the base group. These restrictions are implied by the theory of individual consumer behavior and the requirement that individual demand functions can be aggregated exactly to obtain the aggregate demand functions used in the model. The restrictions are discussed in greater detail in Chapter 2. The estimation of the parameters describing consumer demand from household survey data is also described in Chapter 2.

The demographic characteristics employed in the model include the number of children, the four Census regions, and race, sex and three age groups for the head of household. Since it is infeasible to include demand functions for individual households in IGEM, we create an aggregate version of the demand functions (1.29). To do this we interpret w_k as the vector of full expenditure shares and m_k as full expenditures of a household of type *k*.

Let n_k be the number of households of type k. Then the vector of demand shares for the U.S. economy, $w = (\frac{P_{ND}^{CX} C_{ND}^X}{MF^X}, \frac{P_K^{CX} C_K^X}{MF^X}, \frac{P_{SV}^{CX} C_{SV}^X}{MF^X}, \frac{P_R^{CX} C_R^X}{MF^X})'$, is obtained by aggregating over all types of households:

$$(1.32)\quad w = \frac{\sum_k n_k m_k \mathbf{w}_k}{\sum_k n_k m_k}$$

$$= \frac{1}{D(p)}\left[\alpha^H + B^H \ln p - \mathbf{\iota} B^H \xi^d + B_A \xi^L\right]$$

where the distribution terms are:

$$(1.33)\qquad \xi^d = \sum_k n_k m_k \ln m_k / M\ ; \qquad M = \sum_k n_k m_k$$

$$(1.34)\qquad \xi^L = \sum_k n_k m_k A_k / M$$

For example, the nondurables component of the aggregate share vector is:

$$(1.35)\quad w_{ND} = \frac{P_{ND}^{CX} C_{ND}^X}{MF^X}$$

where MF^X denote the national value of full consumption expenditures in CEX units:

$$(1.36)\quad MF^X = \sum_k n_k m_k = P_{ND}^{CX} C_{ND}^X + P_K^{CX} C_K^X + P_{SV}^{CX} C_{SV}^X + P_R^{CX} C_R^X$$

By constructing an aggregate model of consumer demand through exact aggregation over individual demands, we are able to incorporate the restrictions implied by the theory of individual consumer behavior. In addition, we incorporate demographic information through the distribution terms (1.33) and (1.34). For the sample period we have the actual values of these distribution terms. For the period beyond the sample we project the distribution terms, using projections of the population by sex and race. That is, we project the number of households of type k, n_{kt}, by linking the age and race of the head of household to the projected population. This is explained further in section 1.8 on exogenous projections.

1.2.3.1 Linking the CEX to the NIPAs.

The top tier household model is estimated from data in the Consumer Expenditure Survey (CEX). As explained in Chapter 2, Personal Consumption Expenditures (PCE) in the National Income and Product Accounts (NIPAs), includes many items missed by the Survey. Since we base IGEM on the NIPAs, we must reconcile the CEX-based estimates to the NIPAs-based estimates[6]. We denote the quantity of consumption of item i in the NIPAs by N_i, and the price by PN_i, i=1,2,…35,R, as listed in Table 1.3. That is, PN_i denotes the price taken from the tables for PCE. Corresponding to the nondurables C_{ND} and services C_{SV} commodity groups, we we have the quantities from the NIPAs, N^{ND} and N^{SV}, and their prices, PN^{ND} and PN^{SV}. The price of leisure PN^R is derived by aggregation over the population in (1.56), as explained below.

The equations (1.32) allocate full consumption among the shares w_{ND}, w_K, w_{SV}, and w_R. We denote the shares based on the CEX as $SC_i^X = w_i$. We need to reconcile these to the shares based on NIPAs:

$$(1.37) \quad SC^N \equiv \left(\frac{PN^{ND} N^{ND}}{MF^N}, \frac{PN^K N^K}{MF^N}, \frac{PN^{CS} N^{CS}}{MF^N}, \frac{PN^R N^R}{MF^N} \right)'$$

where MF^N is full consumption. We do this by expressing the difference between the two shares as an autoregressive process:

$$(1.38) \quad \Delta SC_{it} = SC_{it}^N - SC_{it}^X \qquad \text{i=\{ND,K,CS,R\}}$$

$$(1.39) \quad \Delta SC_{it} = \alpha + \beta \Delta SC_{it-1} + \varepsilon_{it} \qquad \varepsilon_{it} = \rho \varepsilon_{it-1} + u_{it}$$

We estimate (1.39) from sample period data and then project it forward. This gives us an exogenous projection of the difference between the two sets of shares and (1.38) gives us the shares SC^N based on the NIPAs.

The value of full consumption in CEX units (1.22) can be rewritten as the sum of the value of leisure and expenditure on commodities:

$$\begin{aligned} MF^X &= P_{ND}^{CX} C_{ND}^X + P_K^{CX} C_K^X + P_{SV}^{CX} C_{SV}^X + P_R^{CX} C_R^X \\ &= P^{CC,X} CC^X + P_R^{CX} C_R^X \end{aligned}$$

After rescaling to NIPA units the value of full consumption is:

$$(1.40) \quad \begin{aligned} PF_t F_t &= P_t^{CC} CC_t + PN^R N^R \\ &= \sum_i PN_{it} N_{it} + PN^R N^R \end{aligned}$$

where $P_t^{CC} CC_t$ denotes the value of aggregate tangible consumption. This is the value that is matched to the Euler equation (1.20) in stage 1.

[6] The estimates discussed in Chapter 2 are entirely based on the CEX, so that data from the NIPAs are not used.

1.2.4 Household model 3rd stage, allocation of demands for commodities.

In the third and final stage of the household model we allocate the quantities of nondurables, capital services, and other services – N^{ND}, N^{K} and N^{CS} – to the 35 commodities, noncompetitive imports and capital services. We do not employ demographic information for this allocation, but utilize a hierarchical model like the one employed for production in Section 1.1. At this stage we impose homotheticity on each of the sub-models.

There is a total of 34 commodity groups, one type of capital services, and one type of leisure, as listed in Table 1.3. These are arranged in 17 nodes, as shown in Table 1.4. This set of nodes is denoted as I_{CNODE} in Appendix A. At each node *m* we represent the demand by a translog indirect utility function, $V^{m}(P^{Hm}, m_m; t)$:

$$(1.41)\quad -\ln V^{m} = \alpha_0 + \alpha^{Hm}\ln\frac{P^{Hm}}{m_m} + \tfrac{1}{2}\ln\frac{P^{Hm}}{m_m}' B^{Hm}\ln\frac{P^{Hm}}{m_m} + f^{Hm}\ln\frac{P^{Hm}}{m_m} \qquad m \in I_{CNODE}$$

$$\ln P^{Hm} \equiv (\ln PN_{m1}, \ldots, \ln PN_{mi}, \ldots, \ln PN_{m,im})' \qquad i \in I_{\mathrm{CNODE}m}$$

The value of expenditures at node *m* is:

$$(1.42)\quad m_m = PN_{m1}N_{m1} + \ldots + PN_{m,im}N_{m,im}$$

The shares of full consumption derived from (1.41) are similar to (1.29), but exclude demographic variables and include latent variables representing changes in preferences f_t^{Hm}. When we impose homotheticity, $\iota' B^{Hm} = 0$, the demands simplify to an expression that is independent of the level of expenditures (m_m):

$$(1.43)\quad SN^{m} = \begin{bmatrix} PN_{m1}N_{m1}/PN^{m}N^{m} \\ \cdots \\ PN_{m,im}N_{m,im}/PN^{m}N^{m} \end{bmatrix} = \alpha^{Hm} + B^{Hm}\ln PN^{Hm} + f^{Hm}$$

With this simplification the indirect utility function reduces to:

$$(1.44)\quad -\ln V^{m} = \alpha^{Hm}\ln P^{Hm} + \tfrac{1}{2}\ln P^{Hm}{}' B^{Hm}\ln P^{Hm} + f^{Hm}\ln P^{Hm} - \ln m_m$$

The first three terms in (1.44) are analogous to the price function (1.4) in the production model. We can define the price of the m^{th} basket as:

(1.45) $\ln PN^m = \alpha^{Hm} \ln P^{Hm} + \frac{1}{2} \ln P^{Hm}{}' B^{Hm} \ln P^{Hm} + f^{Hm} \ln P^{Hm}$

If we also express the value of expenditures as the price (1.45), multiplied by the corresponding quantity:

(1.46) $m_m = PN^m N^m$.

Substituting (1.44) and (1.45) into (1.43), the utility index is the quantity of the m^{th} basket, $V^m = N^m$.

As an example, in the *m=3* node the energy aggregate is a function of N_6 (gasoline), N^{FC} (fuel-coal aggregate), N_{18} (electricity), and N_{19} (gas). The demand shares are:

$$(1.47) \quad SN^{m=3} = \begin{bmatrix} PN_6 N_6 / PN^{m=3} N^{m=3} \\ \cdots \\ PN_{19} N_{19} / PN^{m=3} N^{m=3} \end{bmatrix} = \alpha^{H3} + B^{H3} \ln PN^{H3} + f^{H3}$$

The energy value that appears in the next higher node (*m=2*) is:

(1.48) $PN^{EN} N^{EN} = PN_6 N_6 + PN^{FC} N^{FC} + PN_{18} N_{18} + PN_{19} N_{19}$.

1.2.5 Linking the NIPAs to the Input-Output tables.

The categories of Personal Consumption Expenditures (PCE) from the National Income and Product Accounts (NIPAs) are given in Table 1.3. The expenditures are in purchasers' prices, which include the trade and transportation margins. These prices must be linked to the supply side of the model, where expenditures are in producers' prices. In the official input-output tables this link is provided by a bridge table[7], for example, the PCE expenditures of \$32.9 billion in 1992 for "shoes" is comprised of the following commodity groups from the input-output tables: \$3.7 billion from rubber and plastic, \$11.2 billion from leather, \$0.12 billion from transportation and \$17.8 billion from trade.

Let us denote the bridge matrix by **H**, where H_{ij} is the of commodity i from the input-output accounts in PCE item *j*. The value of total demand by households for commodity *i* is:

[7] For the 1992 Benchmark in the *Survey of Current Business*, November 1997, this is given in Table D, Input-Output Commodity Composition of NIPA Personal Consumption Expenditure Categories.

(1.49) $VC_i = \sum_j H_{ij} PN_j N_j$.

The prices from the NIPAs are also linked to input-output prices through the bridge matrix. Using PS_i to denote the supply price of commodity i (explained in more detail in section 1.5 below), the price of PCE item j is expressed in terms of the transpose of the bridge matrix:

(1.50) $PN_j = \sum_i H_{ij}^T PS_i^C$

The quantity of commodity i consumed by the household sector is: :

(1.51) $C_i = VC_i / PS_i^C \qquad i \in I_{COM}$

The value of total personal consumption expenditures is the sum over all commodities in either definition:

(1.52) $PCC_t CC_t = \sum_i VC_i = \sum_i PN_i N_i$

We emphasize again that the consumption expenditures in IGEM exclude the purchases of new consumer durables but include the service flow from the stock of durables. Purchases of new durables are treated as investment in order to preserve symmetry between housing and consumers' durables.

1.2.6 Other household accounts

The demand for leisure in CEX units is given by the fourth element of the share vector in (1.32). The aggregate demand for leisure in NIPA units (N^R) is obtained by applying the exogenous difference from (1.38). Individual leisure is related to hours supplied to the labor market in (1.26). We construct an aggregate version of this equation by defining the aggregate time endowment LH_t as an index number of the population, where individuals are distinguished by gender, age, and educational attainment. Let POP_{kt} denote the number of people in group k at time t, and the price of time is the after-tax hourly wage of person k, $(1 - tl_t^m) P_{kt}^L$. The value of the aggregate time endowment, allocating 14 hours a day to each person, is:

(1.53) $P_t^h LH_t = VLH_t = \sum_k (1 - tl_t^m) P_{kt}^L * 14 * 365 * POP_{kt}$

We express the value of leisure as the product of the quantity LH and the price of hours P^h. The Tornqvist index for the quantity of the time endowment is:

(1.54) $d\ln LH_t = \sum_k \tfrac{1}{2}(v_{kt}^L + v_{kt-1}^L)d\ln(14*365*POP_{kt})$

$$v_{kt}^L = \frac{(1-tl_t^m)P_{kt}^L*14*365*POP_{kt}}{VLH_t} \qquad \text{k=\{gender,age,education\}}$$

The price of aggregate time endowment is the value divided by this quantity index:

(1.55) $P_t^h = \dfrac{VLH_t}{LH_t}; \qquad P_{baseyear}^h \equiv 1$

In a similar manner we can define the quantity of leisure by aggregating over all population groups, where the annual hours of leisure for a person in group k is denoted by H_{kt}^R:

(1.56) $d\ln N_t^R = \sum_k \tfrac{1}{2}(v_{kt}^R + v_{kt-1}^R)d\ln(H_{kt}^R * POP_{kt})$

$$v_{kt}^R = \frac{(1-tl_t^m)P_{kt}^L * H_{kt}^R * POP_{kt}}{VR_t} \qquad \text{k=\{gender,age,education\}}$$

The leisure hours of group *k* are derived from the accounts for hours worked described in Appendix C. These accounts are used for estimating the quantity of labor input in Section 1.1. Leisure is equal to the annual time endowment, less the average hours worked for individuals in group k:

(1.57) $H_{kt}^R = 14*365 - HH_{kt}$

The value of aggregate leisure is:

(1.58) $VR_t = \sum_k (1-tl_t^m)P_{kt}^L * H_{kt}^R * POP_{kt}$.

The price of aggregate leisure is the value divided by the quantity index:

(1.59) $PN_t^R = \dfrac{VR_t}{N_t^R}; \qquad PN_{baseyear}^R \equiv 1$

It should be pointed out that the price of aggregate time endowment is not the same as the price of aggregate leisure even though at the level of the person in group *k* they are the same. This is due to the differences in aggregation weights and we relate the two with a leisure price aggregation coefficient:

(1.60) $PN_t^R = \psi_{Ct}^R P_t^h$

Taking the aggregation coefficient (1.60) into account, we define labor supply as time endowment, less adjusted leisure:

(1.61) $LS = LH - \psi_C^R N^R$

This implies that the price of leisure is identical to the price of time endowment and the values are related as:

(1.62) $P^h LH = P^h LS + PN^R N^R$

As explained later in Section 1.6, the value of labor supply is related to the payments by employers in the following way:

(1.63) $P^h LS = (1 - tl^m) \sum_j PLD_j LD_j$

Given the labor accounts, we next describe the household financial accounts. In the lifetime budget constraint (1.19), W_0^F represents the present value of the stream of household full income, that is, tangible income plus the imputed value of leisure. Household tangible income, Y_t, is the sum of after-tax capital income (YK^{net}), labor income (YL), and transfers from the government (G^{TRAN}):

(1.64) $Y_t = YK_t^{net} + YL_t + G_t^{TRAN} - TLUMP_t - twW_{t-1}$

Labor income is:

$$(1.65)\quad YL = P^h LS\,\frac{1\text{-}tl^a}{1\text{-}tl^m} = (1-tl^a)\sum_j PLD_j LD_j$$

We distinguish between marginal tax rates and average tax rates. The price of the time endowment and leisure refers to the marginal price, the wage rate reduced by the marginal tax rate, while income is defined in terms of the wage rate less average income taxes.

Capital income is the sum of income from the private stock of physical assets and financial assets in the form of claims on the government and rest-of-the-world:

$$(1.66)\quad YK_t^{net} = (1-tk)YK_t - tpPK_{t-1}K_{t-1} - YK^{gov} + \ (1-tk)\left(GINT_t + Y_t^{row}\right)$$

The components of capital income will be explained in more detail in section 1.3 on capital, section 1.4 on government, and section 1.5 on the foreign accounts. Other items in (1.64) will be discussed in more detail in section 1.4 on the government accounts.

Full income includes the value of the time endowment and is equal to household tangible income Y_t plus the value of leisure:

$$(1.67)\quad YF_t = YK_t^{net} + YL_t + PN_t^N N_t^R + G_t^{TRAN} - TLUMP_t - twW_{t-1}$$

Private household savings is income less consumption, non-tax payments to the government (*TAXN*), and transfers to rest-of-the-world (*CR*) :

$$(1.68)\quad S_t = YF_t - PF_tF_t - CR_t - TAXN_t + tcVCC^{exempt}$$

$$= Y_t - P_t^{CC}CC_t - CR_t - TAXN_t + tcVCC^{exempt}$$

1.3 Investment and the cost of capital

The primary factors of production in IGEM are capital and labor services. Capital includes structures, producer's durable equipment, land, inventories, and consumer durables. This differs from investment in the NIPAs which include purchases of consumers' durables as part of Personal Consumption Expenditures[8]. We focus on capital owned by the private sector in this section; government-owned capital is not part of the capital market and is discussed later. There are two sides to the capital account. The capital stock is rented to the producers described in section 1.1 and the annual rental payment is the capital income of the household sector. The flow of investment is purchased annually to replace and augment the capital stock. We consider both aspects of the capital market.

1.3.1 Aggregate investment and cost of capital

We assume that the supply of capital is determined by past investments; however, capital can may be moved costlessly among industries within any period. We also assume that there are no installation or adjustment costs in converting new investment goods into capital stocks. With these assumptions, the savings decision is identical to the investment decision. We present an alternative approach to the savings-investment decision in order to clarify the role of the cost of capital, a key equation of IGEM.

The owner of the stock of capital chooses the time path of investment by maximizing the present value of the stream of after-tax capital income, subject to a capital accumulation constraint:

$$(1.69)\quad \text{Max} \quad \sum_{t=u}^{\infty} \frac{(1\text{-}tk)(PKD_t \psi^K K_{t\text{-}1}\text{-}tpPK_{t\text{-}1})\text{-}(1 - t^{ITC})PII_t I_t^a}{\prod_{s=u}^{t} 1+r_s}$$

$$(1.70)\quad K_t = (1-\delta)K_{t-1} + \psi^I I_t^a$$

After-tax capital income $(1\text{-}tk)(PKD_t \psi^K K_{t\text{-}1}\text{-}tpPK_{t\text{-}1})$ is related to the YK^{net} term in household income (1.60), and the discount rate r_s is the same as that in the Euler equation (1.20). The stock of capital available at the end of the period is K_t. The rental price of

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[8] Land is in the "fixed, non-reproducible" asset category, and is not part of Investment in GDP (land is transferred, not produced). The rental from land is, of course, included in the income side of GDP.

capital services is PKD_t . We require an aggregation coefficient ψ^K to convert the stock measure to a flow of services.[9] The remaining terms are *tp*, the property tax rate, *tk*, the capital income tax rate, and *PK* the price of the capital stock. Finally, I_t^a is the quantity of aggregate investment and $(1-t^{ITC})PII_t$ is its price net of the investment tax credit. In this version of the model we have ignored tax details such as depreciation allowances and the distinction between debt and equity.

Aggregate investment is a bundle of commodities, ranging from computers to structures. Capital stock is also an aggregate of these commodities, but with different weights. At the level of the individual commodity, the capital accumulation equation is the familiar $KS_{it} = (1-\delta_i)KS_{i,t-1} + I_{it}$. Aggregation over all the commodities results in (1.70) with the coefficient ψ_t^I, an index of aggregate investment quality, and the aggregate depreciation rate, δ .

The solution of the maximization problem gives the Euler equation (see A.3.3 in Appendix A):

(1.71) $$(1+r_t)\frac{(1-t^{ITC})PII_{t-1}}{\psi_{t-1}^I} = (1-tk)(PKD_t\psi_t^K - tpPK_{t-1}) + (1-\delta)\frac{(1-t^{ITC})PII_t}{\psi_t^I}$$

There is a simple interpretation of this equation: If we were to put $(1-t^{ITC})PII_{t-1}$ dollars in a bank in period *t-1* we would earn a gross return of $(1+r_t)(1-t^{ITC})PII_{t-1}$ at *t*. On the other hand, if we used those dollars to buy one unit of investment goods (=ψ^I units of capital) we would collect a rental for one period, pay taxes, and the depreciated capital would be worth $(1-\delta)(1-t^{ITC})PII_t$ in period *t* prices. In a model without uncertainty these two returns are equal.

The assumption of no installation costs implies that new investment goods are perfectly substitutable for existing capital as in (1.70). This means that the price of capital stock is linked to the price of aggregate investment:

(1.72) $$PK_t = \psi_t^{PK} PII_t (1-t^{ITC})$$

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[9] These concepts are explained in Appendix D describing the construction of historical data for investment and capital.

The aggregation coefficient ψ_t^{PK} plays a symmetrical role to ψ_t^I and is used to reconcile the sample period differences in the inflation of these prices. The aggregation coefficients are taken as exogenous in the policy simulations.

In equilibrium the price of one unit of capital stock (*PK*) is the present value of the discounted stream of rental payments (*PKD*). Capital rental prices, asset prices, prices of capital stock, rates of return, and interest rates for each period are related by (1.71). This incorporates the forward-looking dynamics of asset pricing into our model of intertemporal equilibrium. The asset accumulation equation (1.70) imparts backward-looking dynamics.

Combining (1.72) and the Euler equation (1.71), we obtain the well-known cost of capital equation (Jorgenson 1963):

$$(1.73)\quad PKD_t = \frac{1}{(1-tk)}[(r_t - \pi_t) + \delta(1+\pi_t) + tp]PK_{t-1}$$

where $\pi_t = (PK_t - PK_{t-1})/PK_{t-1}$ is the asset inflation rate. The rental price of capital equates the demands for capital by the 35 industries and households with the supply given by K_{t-1}.

As we explain in section 1.6 below, the capital stock K_{t-1} is the supply that meets the demand for capital services from the 35 producers and the household sector. Recall that that the rental payment by industry *j* is PKD_jKD_j. The sum over all industries is the gross income in eq. (1.69):

$$(1.74)\quad PKD_t\psi_t^K K_{t-1} = \sum_j PKD_{jt}KD_{jt}$$

We denote the after-tax total payments by *DIV* (to invoke the notion of dividends). This is gross capital income less the property tax and the capital income tax:

$$(1.75)\quad DIV = (1-tk)[\sum_j PKD_jKD_j - \frac{tk^{hh}PKD_{36}KD_{36}}{1-tk^{hh}} - tpPK_tK_t]$$

Dividend income is the major component of household capital income, YK^{net}, in (1.64) above.

To recapitulate: In IGEM capital formation is the outcome of intertemporal optimization. Decisions today are based on expectations of future prices and rates of return. Policies announced today that affect future prices will affect decisions today.

1.3.2 Investment by commodity

The quantity of total investment demanded by the household/investor in period t is I_t^a when the price is PII_t. In the National Income and Product Accounts this is an aggregate of investment by detailed asset classes – structures, producer durable equipment, consumer durables and inventories. The value of investment by these asset types in 2005 is given in Table 1.5[10]. In the benchmark input-output tables, expenditures in purchaser's prices are linked to producer prices via bridge tables[11]. Using these bridge tables, we have constructed a time series of investment demands by the 35 commodity groups employed in IGEM.

We allocate investment demand I_t^a to the 35 individual commodities by means of a hierarchical tier structure of production models similar to the demand for intermediate inputs in the producer model. At the top tier we express this as a function of fixed and inventory investment:

(1.76) $I^a = I(I^{fixed}, I^{inventory}) = I(I_1, I_2, ..., I_{35})$

We denote the value of private investment by VII:

(1.77) $VII_t = PII_t I_t^a = VII^{\text{fixed}} + VII^{\text{inventory}}$

The demands for intermediate inputs are derived from a hierarchical tier structure of translog price functions in equations (1.10-1.11) and Table 1.2. Similarly, we derive fixed investment commodity demands from a nested structure of investment price functions. That is, we use the price dual to the function, $I^{fixed} = I(IF_1, IF_2, ...IF_{35})$. The tier structure is given in Table 1.6, there is a total of 15 nodes dominated by construction, vehicles and machinery. The set of nodes is denoted by I_{INV}.

The translog price function for node m is a function of the component prices $\{PII_{m1},...,PII_{m,im}\}$ and the latent variables f_t^{Im}. This is written as:

---

[10] This table is updated from Jorgenson, Ho and Stiroh (2005) Table 5.1 which includes information on depreciation rates.

[11] In the 1992 Input-Output Benchmark this bridge table is Table E in the *Survey of Current Business*, November 1997. For example, the $43.6 billion investment in the asset "computers and peripheral equipment" is made up of the following commodities at producer prices: 32.7 from machinery, 3.4 from services, 0.4 from transportation, and 7.1 from trade.

(1.78) $\ln PII^{m} = \alpha^{Im} \ln P^{Im} + \frac{1}{2} \ln P^{Im}{}' B^{Im} \ln P^{Im} + \ln P^{Im}{}' f_t^{Im} + \log \lambda^{I} \qquad m \in I_{INV}$

$$\ln P^{Im} \equiv \left(\ln PII_{m1}, \ldots, \ln PII_{mi}, \ldots, \ln PII_{m,im} \right) \quad i \in I_{INVm}$$

The Kalman filter term f_t^{Im} plays a role identical to that of f_{it}^{node} in (1.10) for the producer model, and is modeled as a VAR like (1.5). The share demands corresponding to the price function are:

(1.79) $SI^{\mathrm{m}} = \begin{bmatrix} PII_{m1} IF_{m1} / PII^{m} IF^{m} \\ \ldots \\ PII_{m,im} IF_{m,im} / PII^{m} IF^{m} \end{bmatrix} = \alpha^{Im} + B^{Im} \ln PII^{Im} + f_t^{Im} \qquad \begin{matrix} \mathrm{m} \in \mathrm{I}_{\mathrm{INV}} \\ \mathrm{mi} \in \mathrm{I}_{\mathrm{INVm}} \end{matrix}$

As an example, in the m=7 node the machinery aggregate is a function of IF_{22} (industrial machinery), IF_{23} (electrical machinery) and IF^{MO} (other machinery). The demand shares are:

(1.80) $SI^{m=7} = \begin{bmatrix} PII_{22} IF_{22} / PII^{m=7} IF^{m=7} \\ PII_{23} IF_{23} / PII^{m=7} IF^{m=7} \\ PII^{MO} IF^{MO} / PII^{m=7} IF^{m=7} \end{bmatrix} = \alpha^{I7} + B^{I7} \ln PII^{I7} + f_t^{I7}$

Total inventory investment is specified as a share of aggregate investment:

(1.81) $VII^{inventory} = \alpha^{IY} VII$

where the share α^{IY} is taken from sample data. Total inventory demand is specified as a Cobb-Douglas function of the commodities, using actual shares in the sample period and the share for the final year of the sample for projections. The value of inventory investment in commodity *i* is:

(1.82) $VII_{\mathrm{i}}^{\mathrm{invy}} = \alpha_i^{IY} VII^{\mathrm{invy}} \qquad i \in I_{\mathrm{COM}}$

The total investment demand for commodity *i* is the sum of the fixed investment and inventory components:

(1.83) $\begin{matrix} VI_i = VI_i^{fixed} + VI_i^{inventory} \\ PS_i I_i = PII_i IF_i + VI_i^{inventory} \end{matrix}$

This set of equations is specified in complete detail in Appendix A, eq. A.3.23.

1.4 Government

The government plays an important role in IGEM. Government spending affects household welfare directly through transfer payments and public health spending and indirectly through public capital that improves private sector productivity. Taxes introduce wedges between prices faced by buyers and sellers and distort the allocation of resources. We do not specify a model for public goods and taxation, but set tax rates exogenously and take the shares of public expenditure by commodity as exogenous. We also set the government deficit exogenously.

A major public activity in the U.S. is the collection of revenues for the social insurance trust funds and the transfers from these funds to households. In the new architecture for the U.S. national accounts discussed by Jorgenson (2009), the trust funds are treated as part of household assets. For example, social security contributions and benefits are regarded as transfers within the household sector and not accounted as government revenue and expenditures. The tax rate on labor income includes federal and state and local income taxes, but not social security contributions.

1.4.1 Government Revenues and Expenditures

The tax codes of the federal, state, and local governments are very complex with progressive rates and numerous deductions. We drastically simplify these codes in order to obtain a tractable representation that captures the key distortions. In IGEM the taxes that are explicitly recognized are sales taxes, import tariffs, capital income taxes, labor income taxes, property taxes, and wealth or estate taxes.

Taxes on production tt_j^{full} are defined in (1.12) and include sales taxes tt_j and environmental taxes. The average tax rate tt_j is chosen to match the revenues collected less subsidies. This is a simplification of the complex combination of federal and state taxes and subsidies on production. The labor taxes discussed in (1.63) in Section 1.2.6 give the effective price of leisure as the price paid by employers after the marginal tax rate tl^m. Similarly, the labor income received as the price after the average tax rate tl^a is given in (1.65). While the income tax code includes standard deductions, progressive rate schedules, alternative minimum taxes, and federal-state interactions, the two labor tax rates captures the key feature that marginal rates are higher than average rates.

The effective capital income tax tk used in (1.73) and (1.75) shows the gap between the payments by producers and receipts by the household. The average tax rate combines the corporate tax with the personal income tax. The property tax tp also appears in the cost of capital equation (1.73); this is mostly state and local property taxes. The wealth tax tw is a deduction from household income in (1.64)[12]. Tariffs tr are described later in (1.97). Non-tax receipts, denoted $TAXN$, include various fees charged by governments and appear as a household expenditure in (1.68). To reiterate: The effective tax rates are chosen to replicate the actual revenues. They are close but not identical to the statutory rates. The construction of effective tax rates is described in more detail in Appendix G.

In addition to taxes that are currently collected we introduce new taxes as part of energy and environmental policy. Environmental taxes may be imposed on unit values or quantities (per dollar or per gallon). The tax on the sales value of industry j's output is tx_j^v, while the unit tax is tx_j^u.[13] Other, non-environmental, unit taxes are denoted tu_j. The result is that the total taxes on a dollar of industry j's output is:

(1.84) $$tt_j^{full} = tt_j + tx_j^v + \frac{tu_j + tx_j^u}{PO_j}$$

This is the full tax on the industry output price PO_j in (1.12).

The model also allows a new consumption tax, that is, a tax on personal consumption expenditures but not on intermediate purchases. We normally assume that the same consumption tax rate applies to all goods, that is, $tc_i=tc$, and this drives a wedge between the supply price given below in (1.99) and the consumer price PS_i^C used in (1.50):

(1.85) $$PS_i^C = (1 + tc_i)PS_i \qquad i \in I_{COM}$$

We also allow for the possibility of a threshold for the consumption tax, an exemption below a certain level, so the actual revenue is:

[12] This tax was known as "estate and gift taxes" in the old NIPA, but is currently accounted as "capital transfer receipts" and is not part of the "net government saving" (current government deficit) calculation, but part of the "net borrowing" account.

[13] See Section 1.7 below on environmental accounting for further details.

(1.86) $R\_CON^{net} = \sum_{I_{COM}} tc_i PS_i C_i - tcVCC^{exempt}$

Finally, we allow an adjustment in taxes that applies only to capital used by the households KD_{36}, for example, changes in mortgage deductibility. This is represented by a revenue item which may be positive or negative:

(1.87) $RK^{hh} = \frac{tk^{hh}}{1-tk^{hh}} PKD_{36} KD_{36}$

In policy simulations we often impose a new tax or subsidy, but wish to maintain revenue neutrality. To accomplish this we subtract a lump sum transfer *TLUMP* from household income in (1.64) and add this to government revenues. New taxes are offset by a negative *TLUMP*. A lump sum transfer could be implemented by increasing the standard deduction under the individual income tax.

Government expenditures fall into four major categories – goods and services from the private sector, transfers to the household and foreigners, interest payments on debt to household and foreigners, and subsidies. The first three are denoted by *VGG*, $G^{TRAN} + GR$ and $GINT + GINTR$. Subsidies are regarded as negative sales taxes and included in the calculation of tt_j in (1.84). Transfers and interest payments are set exogenously, scaled to a preliminary projection of the economy and population in line with the forecasts from the Congressional Budget Office[14]. Total spending on commodities, including labor and capital services, is denoted *VGG* and has to be allocated to individual commodities. Government consumption VG_i of commodity *i* is set to actual purchases in the sample period. For projections it is a fixed share of total spending, using shares from the base year:

(1.88) $VG_{it} = PS_{it} G_{it} = \alpha_i^G VGG_t$

$$PLD_{Gt} LD_{Gt} = \alpha_L^G VGG_t \qquad PKD_{Gt} KD_{Gt} = \alpha_K^G VGG_t = 0$$

The quantity of public consumption G_i is the value divided by the supply price. The government does not rent capital directly from the private sector.

[14] See Section 1.8 below.

1.4.2 Total Government Accounts and Deficits

The total revenue of the government is the sum of all the above-mentioned taxes. Those listed in the following equation are sales tax, tariffs, property taxes, capital income taxes, labor income taxes, wealth taxes, nontax revenues, unit output taxes, externality taxes, consumption taxes, lump sum taxes and income from government enterprises (industry 35):

$$\begin{aligned}R\_TOTAL = {} & R\_SALES + R\_TARIFF + R\_P + R\_K + RK^{hh} + R\_L + R\_W \\ & + TAXN + R\_UNIT + R\_EXT + R\_CON^{net} + TLUMP + YK^{gov}\end{aligned} \tag{1.89}$$

where:

(a) $R\_SALES = \sum_j tt_j PO_j QI_j$

(b) $R\_TARIFF = \sum_i tr_i PM_i M_i$

(c) $R\_P = tpPK_{t-1}K_{t-1}$

(d) $R\_K = tk(\sum_{j=1,36} PKD_j KD_j - RK^{hh}) + tkGINT + tkY^{ROW}$

(e) $R\_L = tl^a P^h LS/(1-tl^m) = tl^a \sum_j PLD_j LD_j$

(f) $R\_W = tw(PK.K + BG + BF)$

(g) $R\_UNIT = \sum_j tu_j QI_j$

(h) $R\_EXT = \sum_j tx_j^v \left(PI_j QI_j + PM_j M_j\right) + \sum_j tx_j^u \left(QI_j + M_j\right)$

(i) $YK^{gov} = (1-tk)PKD_{35}KD_{35}$

R_CON^{net} is eq. (1.86); RK^{hh} is eq. (1.87)

The revenues for the above categories for 2005 are given in Table 1.7; the largest component is labor income taxes.

Total government expenditures are the sum of purchases, transfers and interest payments, and income from government enterprises (industry 35):

$$EXPEND = VGG + G^{TRAN} + GR + GINT + GINTR \tag{1.90}$$

Given our treatment of the social insurance funds as household assets, the government interest payments to domestic bond holders is the sum of the official payments plus payments to social insurance trust funds minus payments from the funds to the government:

(1.91) $GINT = GINT^{p} + GINT^{ss} - TAXSS$

These interest payments are set exogenously as a function of the projected deficits and accumulated government debt. An alternative formulation may be chosen that link the interest rate on the debt to the endogenous rate of return, r_t:

(1.92) $GINT_t = r_t BG_{t-1}$

Government expenditures by these categories are given for 2005 in Table 1.7.

The National Income and Product Accounts (NIPAs) distinguish between current expenditures and investment spending and between current receipts and capital transfers (wealth taxes). This results in a current deficit that is distinct from net borrowing requirement. We do not make this distinction in IGEM and define the public deficit of the entire government as total outlays less total expenditures, a concept equal to the official net borrowing requirement. Denoting the deficit by ΔG we have:

(1.93) $\Delta G_t = EXPEND_t - R\_TOTAL_t$

Deficits add to the public debt. This is separated between debt held by U.S. residents and debt held by foreigners, $BG + BG^{*}$. The construction of these debt series and the other government sector variables is described in Appendix G. In both cases we count only the net debt. The domestic debt is the total deficit less the portion financed by foreigners, $BG_t = BG_{t-1} + \Delta G_t + GFI$.

Unfortunately, the stock of debt is not estimated in the NIPAs, but rather in the Flow of Funds by the Federal Reserve Board. As a consequence the stock of debt is not reconciled with the net borrowing requirement in the NIPAs. For the sample period we introduce a variable for the statistical discrepancy BG^{disc} between the two. The stocks of debt are estimated at market values, so that we adjust for the inflation effect using ΔP_t^{BG}:

(1.94) $BG_t = BG_{t-1} + \Delta G_t + GFI_t + \Delta P_t^{BG} + BG_t^{disc}$

Similarly, the stock of debt to foreigners is the accumulation of government net foreign investment, GFI, plus the inflation adjustment:

(1.95) $BG_t^{*} = BG_{t-1}^{*} - GFI - \Delta P_t^{BG*}$

The deficit, stock of debt and other elements in (1.94) and (1.95) are set to actual values for the sample period. For the projection period the discrepancies are set to zero, the capital gains terms are set to zero, and the deficits are set to official projections as explained in section 1.8.

To summarize: We set tax rates exogenously and set the deficit exogenously. The model generates economic activity and hence revenues are endogenous. Government transfers and interest are set exogenously and thus the remaining item in (1.90), general government spending on goods *VGG*, is determined residually.

1.5 Rest of the world -- exports, imports and total supply

Since IGEM is a one-country model, the supply of goods by the rest of the world (ROW) is not modeled explicitly for each commodity. Similarly, the demand for U.S. exports is not driven by endogenous world growth rates as in multi-country models. We follow the standard treatment in one-country models, that is, we regard imports domestic output as imperfect substitutes, where the elasticity of substitution is not infinite. This is often referred to as the "Armington" assumption and is reasonable at our level of aggregation[15]. We also assume that U.S. demand is not sufficient to change world relative prices.

The total supply of commodity *i* is an aggregate of the domestic and imported varieties:

$$(1.96)\quad QS_{it} = QS(QC_i, M_i, t)$$

Domestic commodity supply, $QC_i = \dfrac{V_i^{QC}}{PC_i}$, is given in (1.17), while M_i denotes the quantity of competitive imports[16]. The price of imports is the world price multiplied by an effective exchange rate *e*, plus tariffs *tr* and, possibly, new externality taxes *tx*:

$$(1.97)\quad PM_{it} = (1 + tr_{it} + tx_i^{rv})e_t PM_{it}^* + tx_i^{ru}$$

where e_t is the world relative price; its role will be made clear after the discussion of the current account balance below. We use the term "landed price" to refer to the border price in dollars, before tariffs:

$$(1.98)\quad PM_{it}^{land} = e_t PM_{it}^*$$

We treat the supply function in a similar manner to the production model given in equations (1.1-1.6). We derive the demand for domestic and imported varieties from a translog price function for the total supply price:

$$(1.99)\quad \begin{aligned} \ln PS_{it} &= \alpha_{ct} \ln PC_{it} + \alpha_{mt} \ln PM_{it} + \tfrac{1}{2}\sum_{ij} \beta_{ij} \ln PC_{it} \ln PM_{it} \\ &\quad + f_{ct}^M \ln PC_{it} + f_{mt}^M \ln PM_{it} \end{aligned}$$

---

[15] That is, while we may regard the imports of steel of a particular type as perfectly substitutable, the output of the entire primary metals industry is a basket of many commodities and would have an estimated elasticity that is quite small.

[16] We have used the notation M_j to denote the inputs of non-energy materials into the industry production function. The distinction from M_i as commodity imports should be clear from the context.

The demands in share form derived from this cost function are:

$$(1.100)\ \frac{PM_{it}M_{it}}{PS_{it}QS_{it}} = \alpha_{mt} + \beta_{mm}\ln\frac{PM_{it}}{PC_{it}} + f_t^{Mi}$$

Again, a $\beta_{mm} = 0$ implies that the demand has unit price elasticity, while a large positive value means an inelastic demand for imports. The total value of the supply of commodity *i* to the domestic market and exports is:

$$(1.101)\ PS_{it}QS_{it} = PC_{it}QC_{it} + PM_{it}M_{it}$$

We have now closed the loop in the flow of commodities. We began with the producer model purchasing intermediate inputs at price PS_i and selling output at price PO_j. The price of intermediates is the price given in (1.99), the price of total supply. We assume that all buyers buy the same bundle of domestic and imported varieties for each type *i*.

Imports into the U.S. have been rising rapidly during our sample period, not only in absolute terms but as a share of domestic output. This change cannot be explained by price movements, so that we employ the Kalman filter approach used for the production and consumption subtiers. The right-hand side of (1.100) contains a latent variable f_{it}^M modeled as a vector auto-regression. The parameters of the import demand function are estimated and reported in section 3.4.

As noted in section 1.2, the inputs into the industry production functions include noncompeting imports. These are goods that do not have close U.S. substitutes, such as coffee. The demands for these goods are derived through a hierarchical tier structure. In the production function (1.9), the value of such imports by industry j is $PNCI_j NCI_j$. The price of these imports is specified like that of competitive imports in (1.97) above:

$$(1.102)\ PNCI_{it} = (1 + tr_{it})e_t PNCI_{it}^*$$

Beyond the sample period, world prices PM_{it}^* are projected to change at the same rate as the productivity growth in U.S. industry prices. The productivity of industry *j*, as represented by the latent term in the cost function (1.4), is assumed to apply to productivity in commodity *i* in the rest of the world. The rate of change of PM_{it}^* is set equal to the change in f_{it}^p.

Exports

The demand for U.S. exports depends on world income and prices. We have two options for modeling export demands. In the first option we write exports of commodity *i* as a function of world income and prices. Since we do not model these endogenously we first make an exogenous projection of world incomes and demands (X_{i0t}). We assume that the world price of commodity *i* falls at the same rate as the change in productivity in the U.S. industry *i* , as projected by the latent term in the industry price function (1.4). With these projections we write the export demand for commodity i as a function of domestic prices and the world prices:

(1.103a) $$X_{it} = X_{i0t}\left(\frac{PC_{it}}{ePM_{it}^{*}}\right)^{\eta_i}$$

In the second option we use a translog share function to allocate total supply between domestic supply and exports. There are data on the actual export prices received in the sample period; in IGEM we use the world price of imports into the U.S., $PM_{it} = e_t PM_{it}^{*}$. We estimate the allocation function in terms of the import price:

(1.103b) $$\frac{PS_{it}X_{it}}{PS_{it}QS_{it}} = \alpha_{xt} + \beta_{xx}\ln\frac{PM_{it}}{PC_{it}} + f_{it}^{X}$$

International balance and foreign assets

The trade balance in dollars is exports less both types of imports:

(1.104a) $$TB_t = \sum_i PC_i X_i - \sum_i e_t PM_i^{*} M_i - \sum_j ePNCI_j^{*} NCI_j$$

The current account balance is the trade balance, plus net interest receipts, and less private and government transfers:

(1.104) $CA_t = TB_t + Y_t^{row} - GINTR - CR_t - GR_t$

The current account surplus, less the portion due to government foreign investment, adds to the stock of net private U.S. foreign assets:

(1.105) $BF_t = BF_{t-1} + CA_t - GFI$

However, there is no consistent set of national accounts for (1.105). Our estimates of the stock of net private US assets require the use of a statistical discrepancy and allow for capital gains:

(1.106) $BF_t = BF_{t-1} + CA_t - GFI + BF^{disc} + \Delta P^{BF}$

The closure of the external sector is treated in various ways in different trade models. We could set the current account exogenously and let the world relative price, e_t, adjust. Alternatively, we could set e_t exogenously and let the current account balance be endogenous. The second option is difficult to implement in a dynamic model where we need to have a clearly defined steady state. We therefore chose the first method, that is, the price of imports and exports move with the endogenous e_t so that (1.104) is satisfied.

1.6 Market balance and intertemporal equilibrium

We have now described all the components of supplies and demands for commodities and factor services. Since IGEM is a dynamic model with intertemporal equilibrium, we have structured it in such a way that the solution algorithm parallels the economic analysis. The economy begins with an initial capital stock and stocks of claims on the government and ROW. The solution algorithm consists of (i) guessing a path of full consumption and capital stocks, K_{t-1}^g and F_t , (ii) calculating the period *t* equilibrium conditional on the initial guesses; (iii) deriving an implied new stock from the period t investment, $K_t^{implied}$, which will be different from K_t^g. Also, the Euler equation (1.20) will not hold for the guessed *F*'s and conditional r_t; (iv) iterating on the guesses until $K_t^g = K_t^{implied}$ and the Euler equation for F_t holds for all t.

We begin by describing the equilibrium in each period, given the initial guess of the capital stock and full consumption. With constant returns to scale and factor mobility the equilibrium prices clear all markets at zero profits for each period. In the commodity markets, the demand side of the economy consists of intermediate demands by producers, household consumption, investor demand, government demand and exports. The supply is given by (1.101). In equilibrium the industry output prices PO_j (and the related PI_j, PC_i, PS_i) move until we have, for each *i*:

(1.107) $$PQ_i\, QS_i = \sum_j PS_i\, QP_{ij} + PS_i(C_i + I_i + G_i) + PC_i X_i$$

In capital market equilibrium, the demand for capital input from all industries (1.6) and households (1.32) is equal to the supply from the stock of inherited capital. In section 1.3 above we have been careful to stress the distinction between the stock and flow of capital. In Appendix D, where we describe the construction of the historical data for capital, we explain how the stock measures are related to the flow of services for each of the asset types, and how these two measures are independently aggregated.[17] As in (1.69), the aggregate supply of services is equal to the stock multiplied by the aggregation coefficient ψ_t^K. The equilibrium condition in value terms is:

(1.108) $$PKD_t\, \psi_t^K K_{t-1} = \sum_j PKD_{jt} KD_{jt}$$

Since we assume that capital is mobile across sectors, there is only one capital rental price needed to clear this market. We observe different rates of return in our historical database for the period 1960-2005. To reconcile the actual movement in historical prices with our simplifying assumption of capital mobility, we treat the industry rental prices as fixed constants times the economy-wide rental price:

(1.109) $$PKD_{jt} = \psi_{jt}^K PKD_t$$

In the sample period we calculate the ψ_{jt}^K coefficients from the actual data on industry costs of capital and the aggregate cost of capital. For the projection period we set these coefficients equal to the last sample point, so that the ratios of marginal products of capital across industries are constant in the projection period. With these industry-specific adjustments, the economy-wide price PKD_t equates supply and demand for capital services:

(1.110) $$\sum_{j=1}^{C} \psi_{jt}^K KD_{jt} = KD_t = \psi_t^K K_{t-1}$$

Turning to the labor market, the supply comes from the household demand for leisure given in (1.61) and the demand is the sum over the 35 industries and government. The equilibrium condition in value terms is:

(1.111) $$P_t^h LS_t = P_t^h \left(LH_t - \psi_{Ct}^R N_t^R \right) = (1 - tl_t^m) \sum_j PLD_{jt} LD_{jt}$$

Recalling the discussion in section 1.2.6, ψ_C^R is an aggregation coefficient linking the time endowment to leisure in a similar manner to the role of ψ_t^K in the capital equation. Also, the time trends of the industry labor prices are quite different among industries.

To reconcile the historical movements of these prices with the simplifying assumption of labor mobility, we set the economy-wide wage rate equal to the price of the time endowment, adjusted for the marginal labor tax. We use fixed constants to scale the industry wage rates to the economy-wide wage rrate:

(1.112) $$PLD_j = \psi_j^L \frac{P^h}{\left(1 - tl^m\right)}$$

The price P^h clears the market for labor input:

(1.113) $$LS_t = LH_t - \psi_{Ct}^R N_t^R = \sum_j \psi_{jt}^L LD_{jt}$$

Three additional equations must hold in equilibrium. The first is the exogenous government deficit (1.93), which is satisfied by the endogenous spending on goods *VGG*, as described in section 1.4.2. The second is the exogenous current account surplus (1.104), which is satisfied by the world relative price, e_t. The third item is the savings and investment relation:

(1.114) $$\begin{aligned} S_t &= P_t^I I_t^a + \Delta G_t + CA_t \\ BG_t &= BG_{t-1} + \Delta G_t + GFI_t + \Delta P_t^{BG} + BG_t^{disc} \\ BF_t &= BF_{t-1} + CA_t - GFI \end{aligned}$$

Household savings is first allocated to the two exogenous items, lending to the government to finance the public deficit and lending to the rest of the world. The remainder is allocated to investment in domestic capital. As we explained in section 1.4, there are no separate savings and investment decisions and (1.114) holds as a result of household intertemporal optimization[18].

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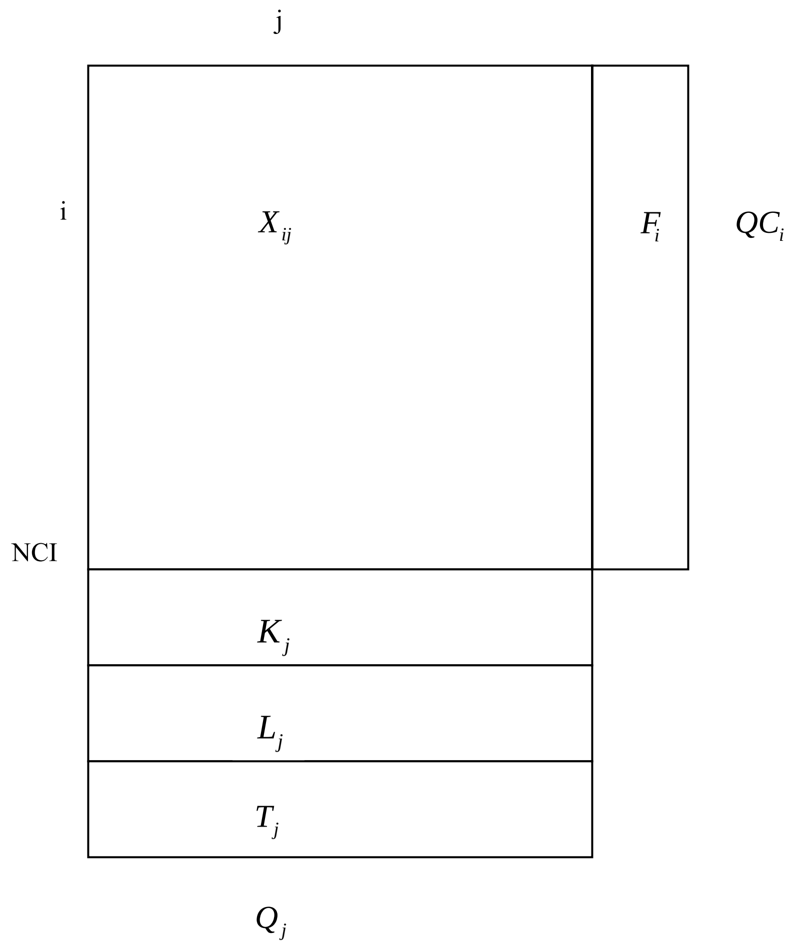
[17] The stocks are aggregated using asset price weights while the service flows are aggregated using the user costs of capital (1.73).

[18] Where investment is derived separately, an endogenous interest rate would clear the equation between saving and investment.

The model is homogenous in prices, so that doubling all prices will leave the equilibrium unchanged. We therefore are free to choose a normalization for the price system and we use the labor price as the numeraire. Furthermore, one of the equations is implied by Walras Law, that is, if n-1 equations hold, the n^{th} will also hold.

The solution algorithm is described in more detail in Appendix I. It uses a hybrid intertemporal algorithm that generalizes Fair and Taylor (1983) and employs certain features of the "multiple shooting" procedure (see Lipton, *et al*., 1982).



Q_j: industry j output
QC_i: quantity of domestic commodity i
K: capital input
L: labor input
T: sales tax
NC: noncompeting imports
X_{ij}: quantity of intermediate input i into j
F_i: final demand for commodity i (C+I+G+X-M)
M_{ji}: quantity of commodity i made by industry j

Fig. 1.1 Input output USE table.

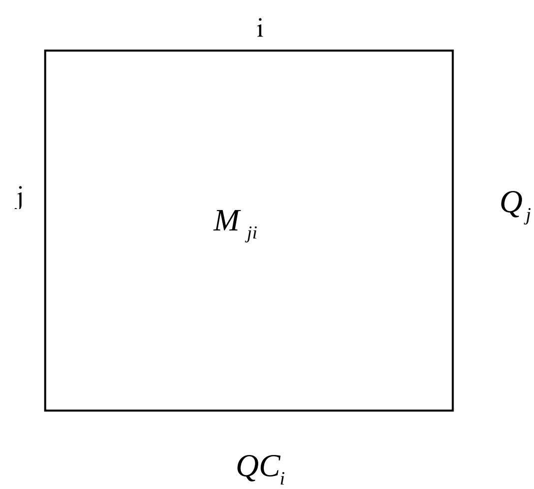


Fig. 1.2 Input output MAKE table.

Table 1.1: Industry Output and Energy inputs, year 2005.

		Coal input ($ml)	Electricity ($ml)	Total Energy ($ml)	Energy share of output (%)
1	Agriculture	0.0	4120.6	18719.3	4.41
2	Metal Mining	15.5	1134.0	2450.5	9.79
3	Coal Mining	1500.6	402.2	3197.2	12.53
4	Petroleum and Gas	0.0	1605.3	19740.8	7.60
5	Nonmetallic Mining	47.2	780.1	2882.9	12.26
6	Construction	0.0	3260.9	36482.9	2.69
7	Food Products	165.4	4507.1	10464.3	1.76
8	Tobacco Products	14.9	72.6	205.8	0.66
9	Textile Mill Products	26.1	1112.8	1943.0	3.23
10	Apparel and Textiles	2.8	280.9	520.9	1.45
11	Lumber and Wood	2.6	1500.7	3727.0	2.88
12	Furniture and Fixtures	14.8	702.5	1958.1	1.93
13	Paper Products	252.1	3514.9	7446.7	4.43
14	Printing and Publishing	0.0	1289.6	2633.3	1.15
15	Chemical Products	263.1	8350.6	25577.8	4.91
16	Petroleum Refining	10.7	4707.1	214660.4	51.25
17	Rubber and Plastic	14.0	3047.6	4649.8	2.47
18	Leather Products	0.9	74.8	170.7	2.69
19	Stone, Clay, and Glass	436.9	2949.2	7660.6	5.92
20	Primary Metals	1160.8	7510.1	12752.2	5.08
21	Fabricated Metals	8.2	3715.0	6448.9	2.18
22	Industrial Machinery and Equipm	11.7	2927.9	5368.2	1.27
23	Electronic and Electric Equipmen	2.9	2718.9	4490.5	1.36
24	Motor Vehicles	34.0	2122.5	3824.5	0.87
25	Other Transportation Equipment	20.0	1524.3	2860.8	1.26
26	Instruments	40.5	1212.9	2056.1	0.99
27	Miscellaneous Manufacturing	1.6	389.5	1114.3	1.84
28	Transport and Warehouse	7.7	7269.7	87450.0	13.09
29	Communications	0.0	2364.8	3997.5	0.76
30	Electric Utilities	14887.2	4611.8	52863.6	14.17
31	Gas Utilities	21.0	204.1	42556.6	54.99
32	Trade	21.5	41705.0	79064.1	3.18
33	FIRE	6.9	24836.7	33751.5	1.23
34	Services	21.0	35390.1	74215.3	1.70
35	Goverment Enterprises	0.0	8717.8	25489.8	7.78

"Energy input" includes feedstocks

Table 1.2 Tier structure of industry production function.

	Sym	Name	Components
1	Q	Gross output	capital, labor, energy, materials Q=f(K,L,E,M)
2	E	Energy	coal mining, petroleum & gas mining, petroleum refining, electric utilties, gas utilities E=f(X3,X4,X16,X30,X31)
3	M	Materials (nonenergy)	Construction, Agriculture Mat, Metallic Mat, Nonmetallic Mat, Services Mat M=f(X6,MA,MM,MN,MS)
4	MA	Agriculture materials	Agriculture, Food manuf, Tobacco, Textile-apparel, Wood-paper MA=f(X1,X7,X8,TA,WP)
5	MM	Metallic Materials	Fab-other metals, Machinery mat, Equipment MM=f(FM,MC,EQ)
6	MN	Nonmetallic Materials	Nonmetal mining, Chemicals, Rubber, Stone, Misc manuf MN=f(X5,X15,X17,X19,X27)
7	MS	Services Materials	Transportation, Trade, FIRE, Services, OS MS=f(X28,X32,X33,X34,OS)
8	TA	Textile-apparel	Textiles, Apparel, Leather TA=f(X9,X10,X18)
9	WP	Wood-paper	Lumber-wood, Furniture, Paper, Printing WP=f(X11,X12,X13,X14)
10	OS	Other services	Communications, Govt. enterprises, NC imports OS=f(X29,X35,X_N)
11	FM	Fab-other Metals	Metal mining, Primary metals, Fabricated metals FM=f(X2,X20,X21)
12	MC	Machinery materials	Ind. Machinery, Electric Machinery MC=f(X22,X23)
13	EQ	Equipment	Motor vehicles, Other transp equip, Instruments EQ=f(X24,X25,X26)

Table 1.3: Personal Consumption Expenditures and leisure, IGEM categories, 2005.

	IGEM categories	Consumption ($bil)	NIPA PCE category
1	Food	719.7	3
2	Meals	449.2	4
3	Meals-Employees	12.3	5,6
4	Shoes	54.9	12
5	Clothing	286.7	14,15,16
6	Gasoline	283.6	75
7	Coal	0.3	40
8	Fuel oil	20.7	40
9	Tobacco	88.3	7
10	Cleaning supplies	137.8	21,34
11	Furnishings	43.3	33
12	Drugs	265.3	45
13	Toys	66.2	89
14	Stationery	19.5	35
15	Imports (travel)	7.3	111
16	Reading	61.4	88,95
17	Rental	333.7	25,27
18	Electricity	133.4	37
19	Gas	64.9	38
20	Water	64.0	39
21	Communications	133.0	41
22	Domestic service	19.9	42
23	Other household	64.4	43
24	Own transportation	262.9	74,76,77
25	Transportation	61.5	79,80,82,83,84,85
26	Medical Services	1350.0	47,48,49,51,55
27	Health Insurance	141.3	56
28	Personal services	115.6	17,19,22
29	Financial services	498.6	61,62,63,64
30	Other services	147.4	65,66,67
31	Recreation	357.8	94,97,98,99,100,101,102,103
32	Education and Welfare	450.9	105,106,107,108
33	Foreign Travel	99.9	110
34	Owner maintenance	202.2	our imputation
35	Durables flow	1972.4	our imputation
	Leisure	14432.3	our imputation

NIPA PCE category refers to the line number in Table 2.5.5 of SCB 2006 August.

Table 1.4 Tier structure of consumption function.

	Sym	Name	Components
1	F	Full consumption	Nondurables, capital, Consumer Services, leisure $F = F(N^{ND}, N_K, N^{CS}, N_R)$
2	ND	Nondurables	Energy, Food, Consumer Goods $N^{ND} = N^{ND}(N^{EN}, N^{F}, N^{CG})$
3	EN	Energy	gasoline, Fuel-Coal, electricity, gas $N^{EN} = N^{EN}(N_6, N^{FC}, N_{18}, N_{19})$
4	F	Food	food, meals, meals-employees, tobacco $N^{F} = N^{F}(N_1, N_2, N_3, N_9)$
5	CG	Consumer goods	Clothing-shoe, Household articles, drugs, Misc goods $N^{CG} = N^{CG}(N^{CL}, N^{HA}, N_{12}, N^{MS})$
6	CS	Consumer Services	Housing, Household operation, Transportation, Medical, Miscellaneous services $N^{CS} = N^{CS}(N^{H}, N^{HO}, N^{TR}, N^{MD}, N^{MI})$
7	FC	Fuel-coal	coal, fuel oil $N^{FC} = N^{FC}(N_7, N_8)$
8	CL	Clothing-shoes	shoes, clothing $N^{CL} = N^{CL}(N_4, N_5)$
9	HA	Household articles	cleaning supplies, furnishings $N^{HA} = N^{HA}(N_{10}, N_{11})$
10	MS	Misc goods	toys, stationery, imports, reading $N^{MS} = N^{MS}(N_{13}, N_{14}, N_{15}, N_{16})$
11	H	Housing services	housing rental, owner occupied maintenance $N^{H} = N^{H}(N_{17}, N_{34})$
12	HO	Household operatio	water, communications, domestic service, other household $N^{HO} = N^{HO}(N_{20}, N_{21}, N_{22}, N_{23})$
13	TR	Transportation	own transportation, transportation $N^{TR} = N^{TR}(N_{24}, N_{25})$
14	MD	Medical	medical services, health insurance $N^{MD} = N^{MD}(N_{26}, N_{27})$
15	MI	Misc services	personal services, Business serv, Recreation, education $N^{MI} = N^{MI}(N_{28}, N^{BU}, N^{RC}, N_{32})$
16	BU	Business services	financial services, other services $N^{BU} = N^{BU}(N_{29}, N_{30})$
17	RC	Recreation	recreation, foreign travel $N^{RC} = N^{RC}(N_{31}, N_{33})$

Table 1.5: Private Investment by Asset Class, 2005 (bil. $).

Total private investment	3101.1
Non-Residential Structures	334.6
Residential Structures	759.2
Equipment and Software	946.5
Household furniture	1.9
Other furniture	36.5
Other fabricated metal products	14.2
Steam engines	4.3
Internal combustion engines	1.2
Farm tractors	10.8
Construction tractors	3.9
Agricultural machinery, ex tractors	10.5
Construction machinery, ex tractors	25.3
Mining and oilfield machinery	7.5
Metalworking machinery	26.0
Special industry machinery, n.e.c.	30.6
General industrial equipment	58.7
Computers and peripheral equipment	89.0
Service industry machinery	19.0
Communication equipment	86.2
Electrical trans., distrib., & industrial app.	21.4
Household appliances	0.2
Other electrical equipment, n.e.c.	6.8
Trucks, buses, and truck trailers	96.7
Autos	31.9
Aircraft	15.0
Ships and boats	4.8
Railroad equipment	4.6
Instruments (Scientific & engineering)	79.9
Photocopy and related equipment	3.6
Other nonresidential equipment	56.0
Other office equipment	6.2
Software	193.8
Consumers Durables	1023.9
Autos	227.1
Trucks	190.8
Other (RVs)	27.0
Furniture	79.9
Kitchen Appliance	36.8
China, Glassware	36.6
Other Durable	85.8
Computers and Software	56.5
Video, Audio	82.7
Jewelry	24.3
Ophthalmic	76.2
Books and Maps	58.4
Wheel Goods	41.8
Land	-
Inventories	36.9

Table 1.6 Tier structure of investment function.

	Sym	Name	Components
	A	Aggregate Investment	Fixed investment, Inventory investment $I^{a} = I(I^{FX}, I^{IY})$
	IY	Inventory	All 35 commodities in flat Cobb-Douglas function VII^{IY}
1	FX	Fixed	Long-lived assets, Short-lived assets $I^{FX} = I(IF^{LG}, IF^{SH})$
2	LG	Long-lived assets	construction, finance-insurance-real estate $IF^{LG} = I(IF_{6}, IF_{33})$
3	SH	Short-lived assets	Vehicles, Machinery, Services $IF^{SH} = I(IF^{VE}, IF^{MC}, IF^{SV})$
4	VE	Vehicles	motor vehicles, other transportation equip. $IF^{VE} = I(IF_{24}, IF_{25})$
5	MC	Machinery	industrial mach, electrical mach, Other Machinery $IF^{MC} = I(IF_{22}, IF_{23}, IF^{MO})$
6	SV	Services	services, Other Services $IF^{SV} = I(IF_{32}, IF^{SO})$
7	MO	Other Machinery	Gadgets, Wood Products, Nonmetallic products, Other Misc. $IF^{MO} = I(IF^{GD}, IF^{WD}, IF^{MN}, IF^{OO})$
8	SO	Other Services	services, Transport-Comm $IF^{SO} = I(IF_{34}, IF^{TC})$
9	GD	Gadgets	primary metals, fabricated metals, instruments $IF^{GD} = I(IF_{20}, IF_{21}, IF_{26})$
10	WD	Wood products	lumber & wood, furniture & fixtures $IF^{WD} = I(IF_{11}, IF_{12})$
11	MN	Nonmetallic produ	chemiclas, rubber, stone-clay-glass, misc. manuf $IF^{MN} = I(IF_{15}, IF_{17}, IF_{19}, IF_{27})$
12	OO	Other misc.	Mining aggregate, Textile aggregate, paper $IF^{OO} = I(IF^{TX}, IF_{13}, IF^{MG})$
13	TC	Transport-Comm	transportation, communications $IF^{TC} = I(IF_{28}, IF_{29})$
14	TX	Textile aggregate	textile, apparel, leather, noncompetitive imports $IF^{TX} = I(IF_{9}, IF_{10}, IF_{18}, IF_{NCI})$
15	MG	Mining aggregate	metal mining, petroleum mining $IF^{MG} = I(IF_{2}, IF_{4})$

Table 1.7 Government revenues and expenditures, 2005

	Variable name	($bil)
Sales tax	R_SALES	450.9
Tariffs	R_TARIFF	34.6
Property tax	R_P	452.3
Capital income tax	R_K	683.3
Labor income tax	R_L	894.1
Wealth tax	R_W	30.3
Nontax revenue	TAXN	74.5
Govt purchases	VGG	2131.7
Transfers to domestic	GTRAN	567.9
Govt transfers to foreigners	GR	27.1
Govt interest to domestic	GINT	158.4
Govt interest to foreigners	GINTR	101.5
Govt deficit (borrowing from persons and SI)	ΔG	506.9
Borrowing from domestic persons and SI	DG	237.5
Borrowing from foreigners (-Govt foreign investment)	-GFI	269.4