THE EFFECTS OF ENVIRONMENTAL REGULATION AND ENERGY PRICES ON U.S. ECONOMIC PERFORMANCE

A thesis presented

by

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Appendix A

A. THE STRUCTURE OF THE MODEL

This appendix describes in detail all of the equations in the model. For a discussion of parameter estimates and exogenous variables, please refer to Appendix B.

A.1. Definitions of Index Sets

It will often be convenient to use the following index sets to describe subscript ranges concisely. First, let the set of demands for a commodity, including both intermediate and final demand, be:

$$DEM = \{1, \ldots, 35, C, I, G, X, M\} \tag{A.1}$$

Similarly, let the set of all possible inputs, including both commodities and primary factors, be given by:

$$INP = \{1, \ldots, 35, N, K, L\} \tag{A.2}$$

where N represents noncompeting imports. Finally, the household model employs commodities based on the National Income and Product Accounts, which differ somewhat from the input-output basis used in the rest of the model. There are 35 such NIPA commodities, and to prevent confusion over whether NIPA or IO commodities are intended, NIPA subscripts will be given as elements of the set *NIPA* below:

$$NIPA = \{1, \ldots, 35\} \tag{A.3}$$

A.2. Relationship of Different Types of Prices

Before describing the model's behavioral equations it's useful to discuss a number of prices that appear in the it, and to specify how they are related. For expositional convenience, the prices will be presented in the order in which they are calculated in the model. Usually this will allow each new price to be expressed solely in terms of exogenous variables or prices that have already been defined, and not in terms of variables not yet presented. The prices will often have subscripts and superscripts; the superscripts denote the type of price, and the subscript indicates a particular instance of that type. For example, P_i^K is the rental price of capital, K, to buyer i. If the subscript is absent, the variable represents the entire vector of prices of that type.

Starting with the prices of primary factors of production, the price of noncompeting imports to buyer $i \in DEM$ is P_i^N, which is the foreign cost multiplied by the exchange rate e and one plus the tariff rate:

$$P_i^N = e \cdot P_i^{NF}(1 + \tau_i^N) \tag{A.4}$$

The rental price of capital to buyers, P_i^K, is the overall rental price of capital multiplied by an aggregation factor:

$$P_i^K = P^{KS} S_i^K \tag{A.5}$$

Although there is only one capital good in the model, the rental price differs across sectors according to the exogenous vector S_i^K.[1] These scale factors arise because the data on which the model is based shows that different industries face different rental prices of capital. This occurs because, in reality, industries do not all buy a single homogeneous capital good. A similar

1. Actually, there are really two capital goods because of the special treatment of the oil sector. This discussion, however, applies to the other 34 industries.

problem arises with the price of labor to different sectors, and another set of scaling variables is used to transform the overall price into sector-specific figures:

$$P_i^L = P^{LB} S_i^L \tag{A.6}$$

The price of labor, in turn, is related to the household's value of time in the following way:

$$P^{LB}(1 - \tau^{LM}) = P^H \tag{A.7}$$

where τ^{LM} is the marginal tax rate on labor, and P^H is the household's value of time. This expression can be used to eliminate the price of labor to produce the following expression for the price of labor to buyers:

$$P_i^L = \frac{P^H}{1 - \tau^{LM}} S_i^L \tag{A.8}$$

As with capital, the scale factors are exogenous. Finally, the household's value of time is the numeraire.

The purchaser's price of industry *i* output is defined in terms of the producer price of output, P_i^O, and the sales tax rate for that industry, τ_i^S. The sales tax is exogenous, but the producer prices are endogenous and will be discussed in detail below.

$$P_i^I = P_i^O (1 + \tau_i^S) \tag{A.9}$$

One of the model's innovations is to incorporate joint production of commodities by all industries. In terms of prices, this means that the price of a particular commodity is not necessarily equal to the price of industry output in the sector for which it is the primary product. For example, construction is produced as a secondary product by many industries, so the price of the construction commodity is not just the price of output of the construction industry. The transformation of industry output into commodities can be regarded as a production process in which industry outputs are the inputs, and commodity outputs are the output. In the case of construction, the construction commodity would be produced out of inputs consisting of the construction output of all industries. For convenience, this function is modelled as Cobb-Douglas, so the commodity prices can be computed as the product of industry output prices, weighted by the share of each industry in total output of the commodity:

$$P_i^C = \prod_j^{35} (P_j^I)^{M_{ji}^C} \tag{A.10}$$

where M_{ji}^C is the share of output of commodity i from industry j in the total value of output of commodity i by all industries. P_i^C is thus the purchaser's price of domestically produced commodity i. Finally, the domestic price of competitive imports of commodity i is just the foreign price of good i multiplied by the exchange rate and one plus the tariff on it:

$$P_i^{IMP} = e \cdot P_i^F (1 + \tau_i^T) \tag{A.11}$$

Both P^F and τ^T are exogenous.

A.3. Substitution Between Imported and Domestic Commodities

Imported and domestic goods are modelled as imperfect substitutes from the point of view of demanders as a whole. That is, all buyers of a good purchase the same composite of imported and domestic production. This substitution is modelled using a translog function of domestic and import prices to determine the aggregate supply price of the commodity. In essence, this is a production function that accepts domestic and imported goods as inputs and produces total domestic supply of the good as an output. Under this interpretation, the derivatives of the function can be used to determine the cost shares of total supply associated with imported and domestic goods, and thus the share of domestic production in total supply.

For convenience, let the vector of imported and domestic prices for commodity i be P_i^{ID}:

$$P_i^{ID} = \begin{pmatrix} P_i^C \\ P_i^{IMP} \end{pmatrix} \tag{A.12}$$

The price of total domestic supply is then given by the translog function:

$$\ln P_i^S = (\alpha_i^F + \frac{\beta_i^{FT}}{1 + e^{-\mu_i^F (t - \tau_i)}})' \cdot \ln P_i^{ID} + \frac{1}{2} \ln P_i^{ID\prime} \cdot \beta_i^F \cdot \ln P_i^{ID} \tag{A.13}$$

This expression is implemented with time-varying parameters in order to capture secular shifts in import demand that seem to be unrelated to price changes. These shifts are easily observed in actual data but are beyond the scope of this model. This relationship can be used to determine the actual prices faced by purchasers of a commodity given the domestic and imported prices. Furthermore, differentiating the equation with respect to the domestic price produces the domestic cost share, W, given by the expression below:

$$W_{ii} = (\alpha_i^F + \frac{\beta_i^{FT}}{1 + e^{-\mu_i^F (t - \tau_i)}} + \beta_i^F \cdot \ln P_i^{ID})_1 \tag{A.14}$$

The subscript 1 on the right hand side is used to indicate that the first element of the vector of shares is the domestic share. It will be convenient later to regard W as a diagonal matrix whose element W_{ii} is the share of domestic production in the total value of supply of commodity i.

A.4. Producer Behavior

Production in each of the 35 industries is represented by a nested translog cost function with constant returns to scale and zero pure profits. Since the industries earn zero profit and have constant returns to scale, it is convenient to substitute the price of output for the unit cost and regard the functions as price frontiers giving the price of industry output for a given vector of industry input prices. There are a total of 38 inputs to each sector: 35 intermediate commodities, and three primary factors, capital, labor and noncompeting imports. Extensive nesting is required because it is not feasible with the data currently available to estimate the unconstrained function. The tier structure used is shown in figure A.1, and summarized in the following table. The formation of each aggregate will be referred to as a "node" of the structure, so there are 13 nodes in all. The top node is capital, labor, energy and materials, while the lower nodes are aggregates of energy goods or materials. This treatment is similar to that used by Goettle and Hudson (1981), which allowed their parameter estimates to be used for the lower nodes. (Refer to Appendix B for details on the sources of all parameters used in the model.)

For all nodes below the KLEM level, the aggregate prices are computed using mostly translog functions of the component prices. All together, there are 420 of these aggregates, 12 nodes by 35 industries. Unfortunately, reliable estimates for about one third of the individual nodes could not be obtained using the translog specification, so these were constrained to be Cobb-Douglas. (Details regarding the parameters used are discussed in Appendix B.) For expositional convenience, all nodes will be discussed in the translog form, although it should be remembered that for some nodes of some industries, the beta matrix will be identically zero.

Figure A.1: The Tier Structure of Production

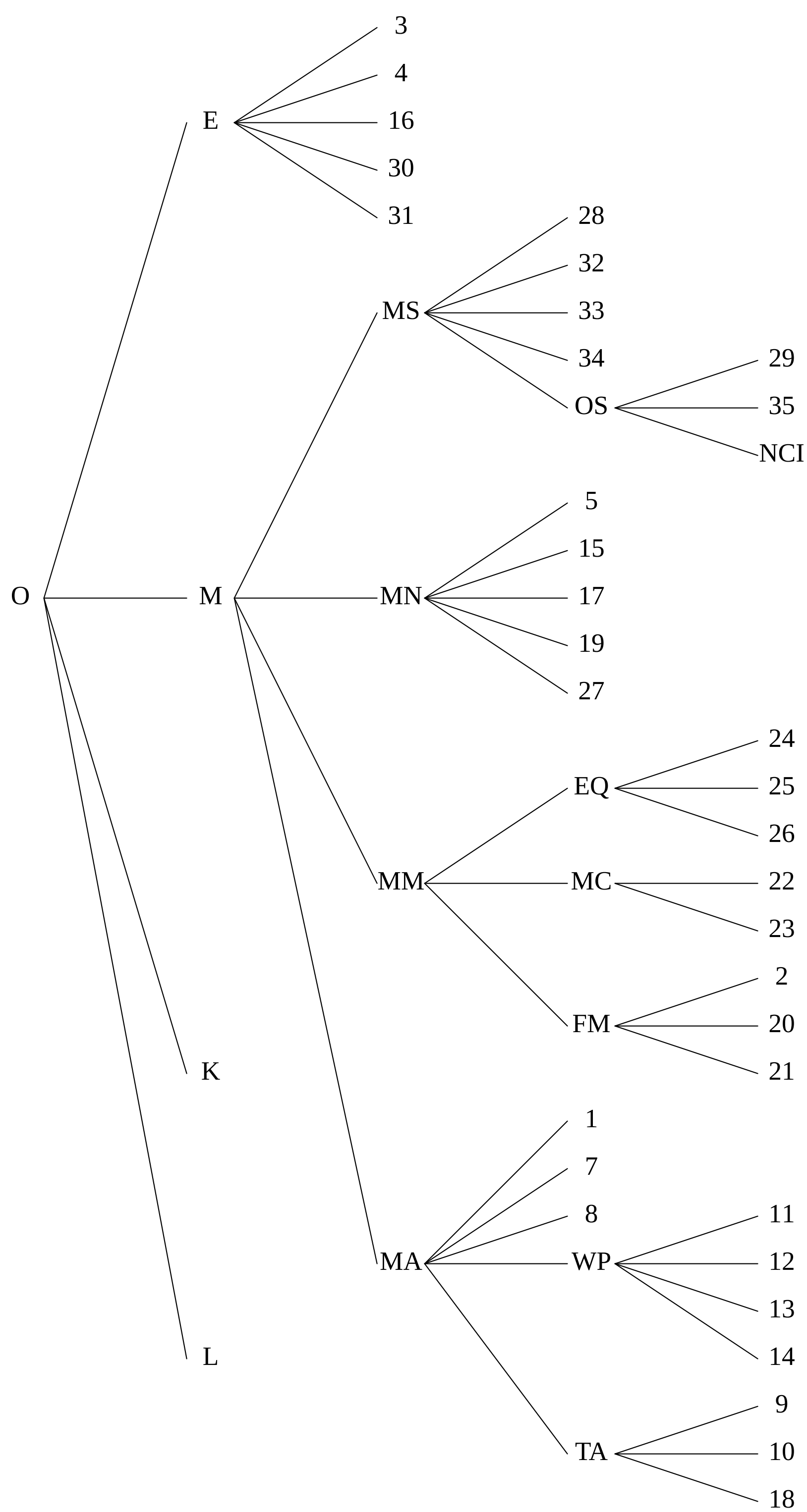

Table A.1: Production Tier Structure

Node	Mnemonic	Interpretation	Components
1	O	Output	K,L,E,M
2	E	Energy	3,4,16,30,31
3	M	Materials & Services	6,MA,MM,MN,MS
4	MA	Agricultural Products	1,7,8,TA,WP
5	MM	Metal Products	FM,MC,EQ
6	MN	Nonmetallic Products	5,15,17,19,27
7	MS	Services	OS,28,32,33,34
8	TA	Textiles & Apparel	9,10,18
9	WP	Wood & Paper Products	11,12,13,14
10	OS	Other Services	29,35,N
11	FM	Primary & Fabricated Metals	2,20,21
12	MC	Machinery	22,23
13	EQ	Equipment	24,25,26

Describing the functions used to define the lower nodes of the structure is best done by example. Consider node 9, which constructs the price aggregate for inputs of wood and paper products. Let the aggregate for industry *i* be denoted P_i^{WP}, and the vector of input prices be P^{IN}. Since the inputs to this node are goods 11, 12, 13 and 14, P^{IN} is as follows:

$$P^{IN} = \begin{pmatrix} P_{11}^S \\ P_{12}^S \\ P_{13}^S \\ P_{14}^S \end{pmatrix} \tag{A.15}$$

Then, the following equation gives price WP for industry *i*:

$$\ln P_i^{WP} = + \alpha_i^{WP\prime} \cdot \ln P^{IN} + \frac{1}{2} \ln P^{IN\prime} \cdot \beta_i^{WP} \cdot \ln P^{IN} \tag{A.16}$$

Although the input prices to the WP aggregate are the same across industries, the parameters α^{WP} and β^{WP} are not. Thus, each industry will have a different value of the price aggregate. Differentiating with respect to logs of input prices yields the vector of cost shares of inputs to this node:

$$W_i^{WP} = \alpha_i^{WP} + \beta_i^{WP} \cdot \ln P^{IN} \tag{A.17}$$

where W_i^{WP} is a vector of cost shares of inputs to the WP node of industry *i*. The other eleven lower nodes are set up in a corresponding way.

The top node−the KLEM level−is specified differently to allow for neutral and biased technical change. Specifically, the price of output is a translog function of the prices of capital, labor, energy, materials and an index of time. If the vector of prices entering the top node of industry *i*'s production function is given by:

$$P_i^{PO} = \begin{pmatrix} P_i^K \\ P_i^L \\ P_i^E \\ P_i^M \end{pmatrix} \tag{A.18}$$

then the price of the industry’s output is given by the expression below:

$$\ln P_i^O = \alpha_i^0 + \alpha_i^{PO\prime} \cdot \ln P_i^{PO} + \frac{1}{2} \ln P_i^{PO\prime} \cdot \beta_i^{PO} \cdot \ln P_i^{PO}$$
$$+\alpha_i^T g(t) + \ln P_i^{PO\prime} \cdot \beta_i^{PT} \cdot g(t) + \frac{1}{2} \beta_i^{TT} g^2(t) \tag{A.19}$$

Neutral technical change occurs through the parameters α_i^T and β_i^{TT}. The presence of the term involving β_i^{PT}, however, allows for biased technical change. That is, the cost shares will change over time, even if prices are constant. When the parameters are estimated, examination of an element of β^{PT} reveals whether technical change in a given industry has been factor-using or factor-saving for that commodity. (For more discussion of this point, including references to other work, refer to Appendix B.) A novel feature of this expression, which will be discussed in detail below, is that time enters through the function $g(t)$. Differentiating with respect to the log of the prices produces the vector of cost shares in the usual way:

$$W_i^{PO} = \alpha_i^{PO} + \beta_i^{PO} \cdot \ln P_i^{PO} + \beta_i^{PT} \cdot g(t) \tag{A.20}$$

In earlier work, such as Jorgenson (1984), $g(t)$ is reduced to simply t. As discussed in Chapter 2, that formulation is unsuitable for this model, which is to be simulated far beyond the end of the sample period used in estimation. The presence of the $\beta_i^{PT} \cdot g(t)$ term will sooner or later drive some of the cost shares negative if g is linear in t. To solve this problem, a logistic function was specified for g, and additional parameters were estimated to produce an individual g

for each industry. In particular, the following function was used:

$$g_i(t) = \frac{1}{1 + e^{-\mu_i(t-\tau_i)}} \tag{A.21}$$

For years much earlier than the estimated τ_i, g will be near zero; for years long after, g will be near one. This means that each industry undergoes a single gradual change in technology. When g is very small, the cost function becomes:

$$\ln P_i^O = \alpha_i^0 + \alpha_i^{PO\prime} \cdot \ln P_i^{PO} + \frac{1}{2} \ln P_i^{PO\prime} \cdot \beta_i^{PO} \cdot \ln P_i^{PO} \tag{A.22}$$

with cost shares given by:

$$W_i^{PO} = \alpha_i^{PO} + \beta_i^{PO} \cdot \ln P_i^{PO} \tag{A.23}$$

Far in the future, when g is near one, technical change will disappear and the cost function will again become invariant with respect to time. At that point, however, the constant and first order terms will have changed:

$$\ln P_i^O = (\alpha_i^0 + \alpha_i^T + \frac{1}{2} \beta_i^{TT}) + (\alpha_i^{PO} + \beta_i^{PT})' \ln P_i^{PO} + \frac{1}{2} \ln P_i^{PO\prime} \cdot \beta_i^{PO} \cdot \ln P_i^{PO} \tag{A.24}$$

with cost shares given by:

$$W_i^{PO} = (\alpha_i^{PO} + \beta_i^{PT}) + \beta_i^{PO} \cdot \ln P_i^{PO} \tag{A.25}$$

During estimation, the parameters are not constrained to produce positive shares as t goes to infinity, but the estimates do, in fact, have this property, so the use of a logistic function for $g(t)$ eliminates the pathological properties of the ordinary trend specification.[2]

Once all the price aggregates and value shares have been evaluated for a particular sector, it is possible to determine the industry's input-output coefficients. This is accomplished by multiplying out the nested shares to produce shares in total output, and hence input. For example, to find the share of input accounted for by sector 21, fabricated metal products, the share of materials in output would be multiplied first by the share of metal products in materials, then by the share of primary and fabricated metals in metal products, and finally by the share of fabricated metals in primary and fabricated metals. The result would be the input-output coefficient for fabricated metal products into the specified industry. This method was used to compute all of the 1225 input-output coefficients in the model. Because the tier structure is fairly complicated, it is difficult to formulate this process concisely in algebra. About the best that can be said is that if W_{ji}^{IO} is a vector whose length n is the depth in tiers of a particular input, and whose elements are the shares along the path down the production structure, then the input-output coefficient for inputs of good i into industry j is:

$$IO_{ji} = \prod_{s=1}^{n} (W_{ji}^{IO})_s \tag{A.26}$$

2. Unfortunately, it was still possible for large swings in prices to drive the cost shares negative, particularly where the share was initially small and the corresponding elements of the beta matrix were large. When this occurred during simulations, the offending share was set to zero and the other scaled back to sum to one.

A.5. Household Behavior

The household model is based a time-separable intertemporal utility function of the following form[3]:

$$U = \sum_{t=0}^{\infty} N_0 \prod_{s=1}^{t} \left(\frac{1 + n_s}{1 + \rho} \right) \cdot \ln F_t \tag{A.27}$$

where F_t is a per capita aggregate of goods and leisure consumed in period t, ρ is the time preference rate, N_0 is the initial population, and n_s is the population growth rate in period s. (The variable F_t will often be referred to as "full consumption", since it includes both consumption of goods and leisure.) The household's decision is subject to the following budget constraint, which requires the present value of the path of consumption to be equal to the present value of wealth:

$$FW_0 = N_0 P_0^F F_0 + \sum_{t=1}^{\infty} N_0 P_t^F F_t \cdot \prod_{s=1}^{t} \left(\frac{1 + n_s}{1 + r_s} \right) \tag{A.28}$$

The first order conditions can be found by forming the Lagrangian and differentiating with respect to F at a particular time t_1. Comparing the resulting expression with that for another time, t_2, produces the Euler equation shown below, which relates the optimal full consumption in the two periods:

$$P_{t_2}^F F_{t_2} = P_{t_1}^F F_{t_1} \prod_{s=t_1+1}^{t_2} \left(\frac{1 + r_s}{1 + \rho} \right) \tag{A.29}$$

When $t_2 = t_1 + 1$, this expression reduces to the more familiar one shown below:

3. This specification is similar to that used by Yun (1984).

$$P^F_{t_1+1}F_{t_1+1} = P^F_{t_1}F_{t_1}\left(\frac{1+r_{t_1+1}}{1+\rho}\right) \tag{A.30}$$

As discussed in Appendix E, however, it will be convenient during numerical solution of the model to use the multiperiod version of the equation. The equations above are both in terms of per capita consumption, but the equivalent versions for aggregate consumption can be derived using the following relationship:

$$F^A_t = F_t N_0 \prod_{s=1}^{t}(1+n_s) \tag{A.31}$$

where F^A_t is aggregate full consumption. This produces the version of the Euler equation actually used in the model:

$$P^F_{t_2}F^A_{t_2} = P^F_{t_1}F^A_{t_1}\prod_{s=t_1+1}^{t_2}\frac{(1+n_s)(1+r_s)}{1+\rho} \tag{A.32}$$

This expression summarizes the consumer's intertemporal optimization completely. Moreover, it captures all household expectations through the value of full consumption, $P^F_{t_2}F^A_{t_2}$. A path of F that satisfies this relationship at every point in time can be said to be a "perfect foresight" trajectory in that household expectations will be fulfilled.[4]

The intraperiod household model, in turn, allocates full consumption to leisure and goods, and further divides goods expenditure among actual commodities. Full consumption is taken to

4. Substantial numerical effort is required to compute a perfect foresight equilibrium, and how it was accomplished is discussed in detail in Appendix E. Roughly speaking, a special algorithm guesses the path of full consumption and computes a solution to the model conditional on it. This produces values which can be used to revise the guess, to bring it closer to the perfect foresight path. Eventually the process converges, and household expectations are fulfilled.

be a translog function of consumption and leisure with time-varying parameters. Specifically, the logarithm of the price of full consumption is given by:

$$\ln P^F = \alpha_0^H + (\alpha^{CL} + \frac{\beta_{CLT}}{1 + e^{-\mu^H(t-\tau^H)}}) \cdot \ln P^{CL} + \ln P^{CL}{}' \cdot \beta^{CL} \cdot \ln P^{CL} \quad \text{(A.33)}$$

where

$$P^{CL} = \begin{pmatrix} P^{CC} \\ P^{LEIS} \end{pmatrix} \quad \text{(A.34)}$$

P^{CC} is the price of an aggregate consumption good, and P^{LEIS} is the price of leisure. Computing the price of leisure is straightforward: it is the value the household places on an hour of time multiplied by a scale factor arising from construction of the data (see Appendix B):

$$P^{LEIS} = P^H S^{LEIS} \quad \text{(A.35)}$$

The construction of P^{CC} is more involved, and will be addressed in detail below.

The details of the parameter estimates are presented in Appendix B, but it is worthwhile to mention briefly why the formulation above was chosen. By allowing the first order terms to vary over time, it was possible to account for the rapid entry of women into the labor force during the 1970's and 1980's. The effect can be seen clearly in the share equations derived by differentiating the full consumption price function with respect to the input prices:

$$\omega^{CL} = \alpha^{CL} + \frac{\beta^{CLT}}{1 + e^{-\mu^H(t-\tau^H)}} + \beta^{CL} \cdot \ln P^{CL} \quad \text{(A.36)}$$

The presence of the β^{CLT} term means that even if relative prices had been constant for all time, the shares of consumption and leisure in full consumption would have changed from α^{CL} far in the past to $\alpha_{CL} + \beta^{CLT}$ far in the future. The estimated value of β^{CLT} for leisure is negative, so the share of leisure consumed decreases over time. Because the time of women who don't work is counted as leisure, this corresponds well with the entry of women into the workforce.

Once these shares have been determined, the value of consumption expenditure can be found:

$$CE = \omega_1^{CL} \cdot P^{FC} FC \tag{A.37}$$

Using this it is straightforward to calculate the quantity of leisure consumed:

$$LEIS = \frac{P^{FC} \cdot FC - CE}{P^{LEIS}} \tag{A.38}$$

To compute the price of the aggregate consumption good, it is necessary to solve the household's expenditure allocation problem. The formulation used here is based on the exact aggregation approach of Jorgenson, Lau and Stoker (1980,1982), and Jorgenson and Slesnick (1987). There are 672 different consumer groups each buying five aggregate goods: energy, food, consumer goods, capital services, and consumer services. Each consumer group has a translog indirect utility function which depends on a vector of attributes A_k of the consumer and has the form:

$$V_k = F(A_k) + \ln \frac{P}{M_k}{}'\alpha^H + \frac{1}{2} \ln \frac{P}{M_k}{}'\beta^{HH} \ln \frac{P}{M_k} + \ln \frac{P}{M_k}{}'\beta^{HA} A_k \tag{A.39}$$

where P is a vector of prices and M_k is the total expenditure of consumer group k. Specifically, P is the following:

$$P = \begin{pmatrix} P^{EN} \\ P^{F} \\ P^{CG} \\ P^{KS} \\ P^{CS} \end{pmatrix} \tag{A.40}$$

The attributes vector, A_k, is derived from the classification of consumers into groups, and has sixteen elements. Its construction is discussed in Appendix B.

The expenditure shares can be derived by using Roy's Identity in logarithmic form:

$$\omega_{ik} = \frac{\dfrac{\partial \ln V_k}{\partial \ln(P_i/M_k)}}{\sum_{j=1}^{n} \dfrac{\partial \ln V_k}{\partial \ln(P_j/M_k)}} \tag{A.41}$$

where ω_{ik} is the share of group k's expenditure devoted to commodity i. Applying this to the indirect utility function produces consumer group k's expenditure shares:

$$\omega_{ik} = \frac{1}{D(P)} \left(\alpha^H + \beta^{HH} \cdot \ln(P/M_k) + \beta^{HA} \right) \tag{A.42}$$

where

$$D(P) = \sum_{i=1}^{n} (\alpha^H + \beta^{HH} \ln(P/M_k) + \beta^{HA} \cdot A_k) \tag{A.43}$$

Imposing the restrictions implied by exact aggregation and integrability (Jorgenson, Lau and Stoker, 1982) requires the following hold:

$$\iota' \beta^{HH} \cdot \iota = 0 \tag{A.44}$$

$$\iota' \beta^{HA} = 0 \tag{A.45}$$

$$\iota \cdot \alpha^{H} = -1 \tag{A.46}$$

where ι is a vector of ones, and α^H has been normalized to -1 for convenience. These imply that the denominator of the expenditure share equation may be rewritten:[5]

$$D(P) = -1 + \iota' \beta^{HH} \cdot \ln P \tag{A.47}$$

Aggregate expenditure shares can be obtained by adding up the weighted sum of expenditure shares over consumer groups, using the groups' expenditures as weights:

$$\omega_i = \frac{1}{D(P)} \left(\alpha^H + \beta^{HH} \ln(P/M^*) + \beta^{HA} \cdot A^* \right) \tag{A.48}$$

where M^* and A^* are defined as follows:

$$\ln M^* = \frac{\sum_{k=1}^{n} M_k \ln M_k}{\sum_{k=1}^{n} M_k} \tag{A.49}$$

$$A^* = \frac{\sum_{k=1}^{n} M_k A_k}{\sum_{k=1}^{n} M_k} \tag{A.50}$$

5. See Jorgenson, Lau and Stoker (1982) for details.

These variables depend on the details of the distribution of expenditure. For tractability during simulation, the shares of consumer groups in total expenditure are assumed to be constant. In particular, the expenditure of group k is:

$$M_k = \lambda_k \cdot M \tag{A.51}$$

This amounts to assuming that all groups own assets in the same proportions, but in varying total amounts. This assumption allows the two expressions above to be rewritten as follows:

$$\ln M^* = \ln M + \frac{\sum_{k=1}^{n} \lambda_k \ln \lambda_k}{\sum_{j=1}^{n} \lambda_j} = \ln M + \lambda^M \tag{A.52}$$

$$A^* = \frac{\sum_{k=1}^{n} \lambda_k A_k}{\sum_{j=1}^{n} \lambda_j} = \lambda^A \tag{A.53}$$

Appendix B describes the source of data on λ_k. Since it was exogenous, λ^M and λ^A could be computed directly. λ^A has a useful interpretation: each element of it is the share of total expenditure accounted for by individuals possessing that attribute. In contrast, λ^M is a statistic of the income distribution.

Parameter estimates for the aggregate share equations were obtained from Jorgenson and Slesnick (1987). These, however, were estimated using National Income and Product Accounts commodity definitions. Unfortunately, these are not the same as the input-output basis commodities used elsewhere in the model. Converting from one basis to the other was accomplished using a bridge table taken from the 1977 benchmark input-output study. The table shows for each NIPA commodity the values of the IO commodities which compose it. Under the assumption that NIPA commodities are Cobb-Douglas composites of IO commodities, it is straightforward to compute the prices of NIPA goods given this table and the supply prices of IO goods. If IN is an array whose element IN_{ij} is the share of the value of NIPA commodity j contributed by IO commodity i, then the price of consumption commodity j is given by the following:

$$P_j^{NIPA} = \prod_{i \in INP} (P_i^S)^{IN_{ij}} \tag{A.54}$$

Since the basic NIPA commodity prices don't enter the top level expenditure share equations directly, it was necessary to estimate a nested tier structure relating the top level price aggregates to actual commodities. The tier structure actually used is displayed in figure A.2, and summarized in the following table. The structure was designed to provide maximum flexibility between inputs while keeping the number of goods in each node to five or fewer. This produced the relatively flat structure shown in the figure. The elements of the nodes were chosen to place close substitutes together. In a sense, the tier structure was designed to maximize the second order coefficients in the expenditure share equations of each node. That is, to put together commodities for which there were strong substitution or complementarity effects.

For nodes below the top level, the consumer group differences were ignored, allowing a simpler price aggregation function to be used. These functions can be illustrated most clearly by an example. Consider the formation of the energy price aggregate, P^{EN}. Let the vector of component prices be:

$$P_{IN}^{EN} = \begin{pmatrix} P^{FC} \\ P_6^{NIPA} \\ P_{18}^{NIPA} \\ P_{19}^{NIPA} \end{pmatrix} \tag{A.55}$$

Figure A.2: The Tier Structure of Consumption

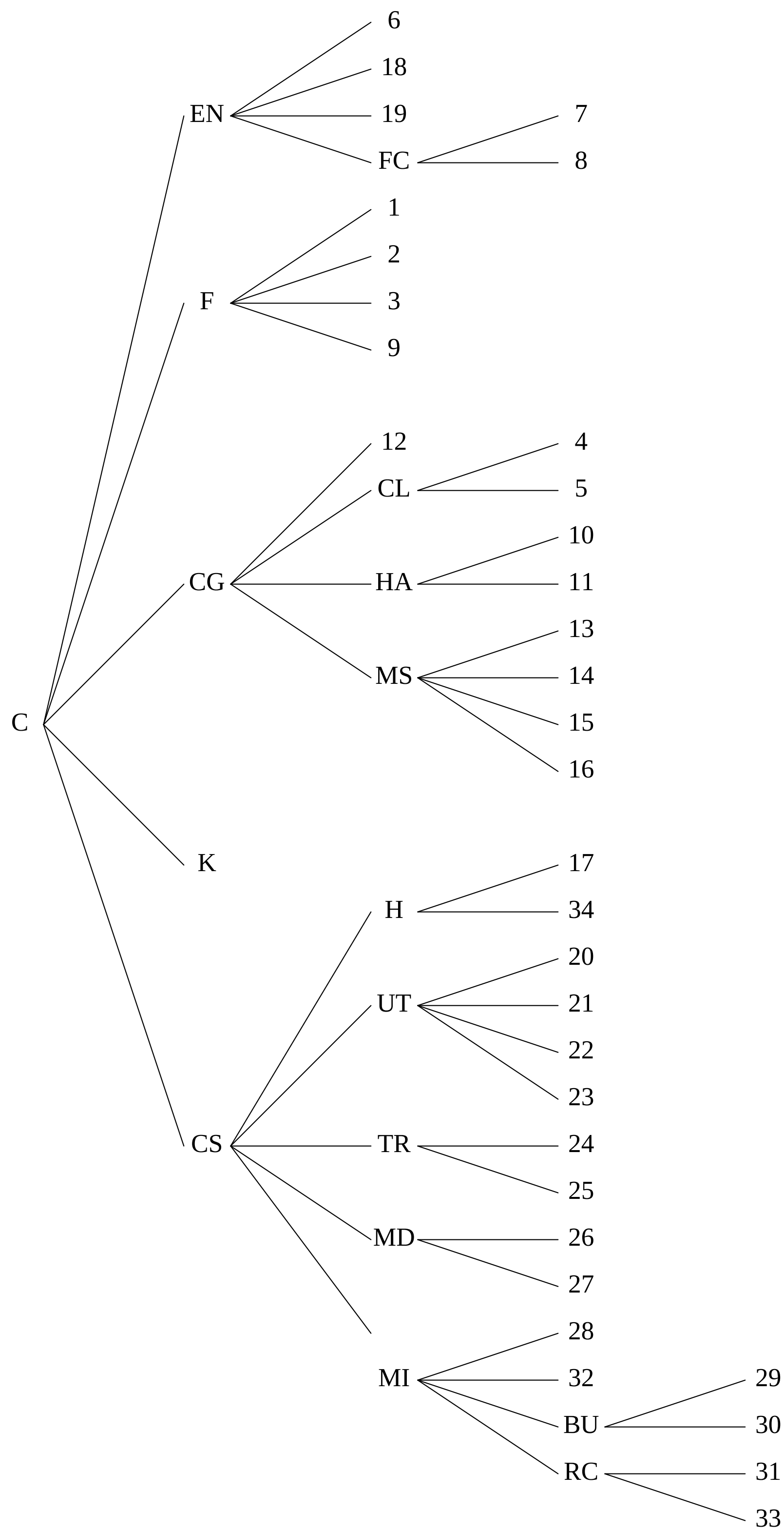

Table A.2: Consumption Tier Structure

Node	Mnemonic	Interpretation	Components
1	CC	Total Consumption	EN,F,CG,P_C^K,CS
2	EN	Energy	FC,6,18,19
3	F	Food	1,2,3,9
4	CG	Consumer Goods	CL,HA,12,MS
5	CS	Consumer Services	H,HO,TR,MD,MI
6	FC	Fuel & Coal	7,8
7	CL	Clothing & Shoes	4,5
8	HA	Household Articles	10,11
9	MS	Miscellaneous Goods	13,14,15,16
10	H	Housing	17,34
11	HO	Household Operation	20,21,22,23
12	TR	Transportation	24,25
13	MD	Medical	26,27
14	MI	Miscellaneous Services	28,BU,RC,32
15	BU	Business Services	29,30
16	RC	Recreation	31,33

Then, the price of the energy aggregate is given by:

$$\ln P^{EN} = \alpha^{EN} \cdot \ln P_{IN}^{EN} + \frac{1}{2} \ln P_{IN}^{EN} \cdot \beta^{EN} \cdot \ln P_{IN}^{EN} \quad \text{(A.56)}$$

where α^{EN} and β^{EN} are parameters estimated as described in Appendix B. Unlike the top node, the parameters are normalized so that α^{EN} sums to 1. Differentiating with respect to component prices produces the expenditure shares of the input commodities:

$$\omega^{EN} = \alpha^{EN} + \beta^{EN} \cdot \ln P_{IN}^{EN} \quad \text{(A.57)}$$

Once the shares at each node have been calculated, the final expenditures shares for each commodity can be computed by multiplying out the shares between the commodity and the top node in a manner similar to that used for the production model. For example, the share of expenditure devoted to coal is the product of the share spent on energy, the share of energy going to coal and oil, and the share of coal in coal and oil.

After the expenditure shares are known, the value of each NIPA commodity consumed can be determined using the total value of spending:

$$CON_i^{NIPA} = \omega_i^C \cdot CE \quad \text{(A.58)}$$

where ω_i^C is the share of NIPA commodity i in total consumption. Then, the consumption of IO commodities can be found by using the bridge table discussed earlier. Since IN_{ij} is the share of IO commodity i in the value of NIPA commodity j, the total consumption of IO commodity i is given by the following:

$$CON_i = \sum_{j \in NIPA} IN_{ij} \cdot CON_j^{NIPA} \tag{A.59}$$

This produces the final demand column for consumption.

Finally, it is necessary to specify how the price of aggregate consumption can be computed, since it is not implied by the model of household behavior described so far. The function used was a modified version of the translog social cost of living index developed by Jorgenson and Slesnick (1983). Specifically, the aggregate price of consumption of goods in year t relative to the price in 1982 is given by:

$$\ln P_t^{CC} = (1 + D(P_t)) \cdot \ln \left(\frac{M_t}{\sum_k N_t^{HH} \omega_k m_0(P_t, A_k)} \right) - \ln P_t{}'(\alpha^H + \frac{1}{2} \beta^{HH} \cdot \ln P_t) \tag{A.60}$$

where

$$D(P_t) = -1 + \iota' \beta^{HH} \cdot \ln P_t \tag{A.61}$$

N_t^{HH} is the total number of households in year t, and ω_k is the fraction of total households accounted for by consumer group k.

A.6. Investment

Investment goods are produced out of commodities according to a special production function. As with production and consumption, a nested tier structure is used, as shown in figure A.3. At the top, total investment is formed by combining inventories and fixed investment. Fixed investment itself is determined by a nested tier structure of translog cost functions, as summarized

Figure A.3: The Tier Structure of Fixed Investment

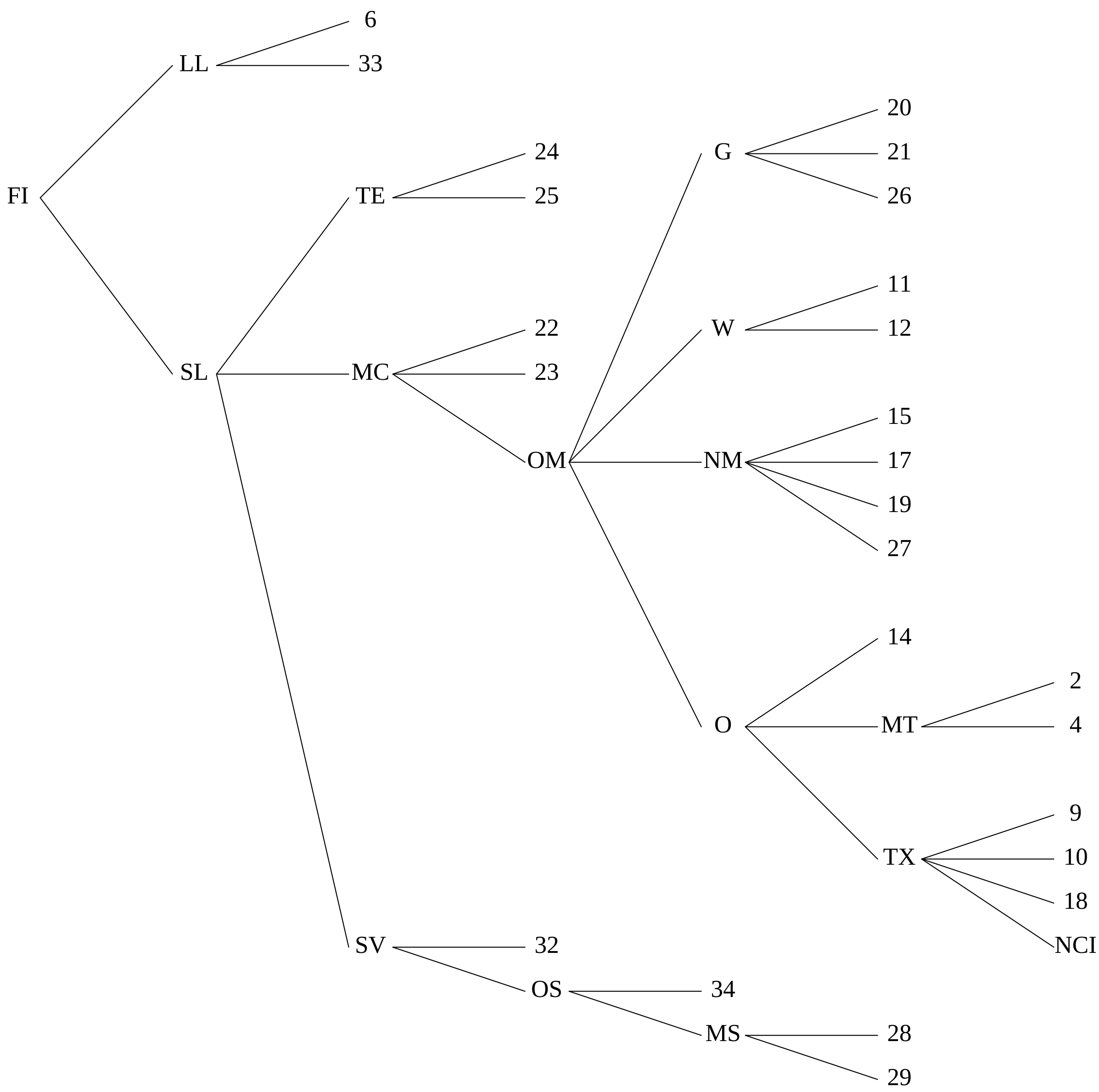

Table A.3: Investment Tier Structure

Node	Mnemonic	Interpretation	Components
1	FI	Total Fixed Investment	LL,SL
2	LL	Long-Lived	6,33
3	SL	Short-Lived	TE,MC,SV
4	TE	Transportation	24,25
5	MC	Machinery	22,23,OM
6	SV	Services	32,OS
7	OM	Other Machinery	G,W,NM,O
8	OS	Other Services	34,MS
9	G	Gadgets	20,21,26
10	W	Wood	11,12
11	NM	Nonmetal	15,17,19,27
12	O	Other	TX,14,MT
13	MS	Movers	28,29
14	TX	Textiles	9,10,18,36
15	MT	Mining	2,4

in the table below. Several commodities are missing from the table as they are not used for investment goods: 1, 3, 5, 7, 8, 13, 16, 30, 31, 35. As with the consumption tiers, goods were grouped together on the basis of substitutability. A more complete discussion of this can be found in Ho (1988b).

As in the cases of production and consumption, it is easiest to describe the tier structure by example. Consider node number 5, which forms the machinery aggregate out of machinery, electrical machinery and other machinery. If the vector of prices of the components of the node is P^{IN}:

$$P^{IN} = \begin{pmatrix} P^S_{22} \\ P^S_{23} \\ P^{OM} \end{pmatrix} \tag{A.62}$$

then the output price will be given by the following:

$$P^{MC} = \alpha^{MC\prime} \ln P^{IN} + \frac{1}{2} \ln P^{IN\prime} \beta^{MC} \cdot \ln P^{IN} \tag{A.63}$$

where α^{MC} and β^{MC} are parameters estimated as described in Appendix B. The share of commodities in the composition of the aggregate are obtained by differentiating the price function to produce:

$$\omega^{MC} = \alpha^{MC} + \beta^{MC} \cdot \ln P^{IN} \tag{A.64}$$

As with the other nested tier structures, the overall shares of commodities in the total are obtained by multiplying out the shares at different tiers. This will produce a vector of commodity shares which can be called ω^{FI}.

Finally, notice that unlike the consumption and production models, the top node of the investment model is specified exactly like the lower nodes. This means that the price of the fixed investment aggregate is just the following:

$$P^{II} = \alpha^{II\prime} \ln P^{IN} + \frac{1}{2} \ln P^{IN\prime} \beta^{II} \cdot \ln P^{IN} \tag{A.65}$$

where P^{IN} is a two element vector consisting of the prices of long and short-lived assets. This price differs from the price of new capital goods by an aggregation constant (see Appendix B). The actual price of new capital goods is

$$P^{K} = S^{PK} \cdot P^{II} \tag{A.66}$$

where S^{PK} is a constant arising from the method used to construct the model's basic data.

Total investment is divided into fixed and inventory investment using an exogenous share determined from the data. That is, the value of fixed investment is the amount of total investment not being devoted to inventories:

$$INV^{F} = INV \cdot (1 - \omega^{INVEN}) \tag{A.67}$$

This implies that the value of inventory investment is just:

$$INV^{I} = INV \cdot \omega^{INVEN} \tag{A.68}$$

Total inventory investment is divided among commodities according to an exogenous vector of shares, ω^{II}. Combining this with the shares of commodities in fixed investment produces the total value of commodities demanded for investment. For commodity i, this would be the following:

$$INV_i = INV^F \cdot \omega_i^{FI} + INV^I \cdot \omega_i^{II} \tag{A.69}$$

Once this vector has been computed, the final demand column for investment has been completely determined.

A.7. Government

Because it would be difficult to determine the appropriate objective function, government behavior is not derived from optimization. Instead, the government follows a simple rule: it collects taxes, runs an exogenous deficit or surplus, and spends its resulting income on goods and services in a pattern determined exogenously. Furthermore, no distinction is made between the federal, state and local levels. Proceeding through the revenue side first, the total revenue from sales taxes is:

$$R^S = \sum_{i=1}^{35} IND_i \tau_i^S \tag{A.70}$$

The revenue from tariffs on competing imports is:

$$R^T = \sum_{i=1}^{35} IMP_i \frac{\tau_i^T}{1 + \tau_i^T} \tag{A.71}$$

Tax revenue from property taxes is given by:

$$R^P = \tau^P V^K \tag{A.72}$$

Tax revenue from wealth taxes is:

$$R^W = \tau^W FW \tag{A.73}$$

Because of data constraints (discussed in Appendix B), a single aggregate tax is levied on capital income. The rate is an amalgamation of the corporate and personal income tax rates weighted by the fraction of capital owned by the corporate and noncorporate sectors. Moreover, a proportional tax is used, so there is no distinction between marginal and average rates. This formulation was necessary because the model has only one capital stock, so an aggregate tax rate was required. Thus, the revenue from capital taxes is:

$$R^C = \tau^C KI \tag{A.74}$$

For labor, however, marginal and average rates are distinguished. Tax revenue depends on the average rate, τ^{LA}, while labor supply (discussed elsewhere) is a function of the marginal rate τ^{LM}. This means that labor tax revenue is just:

$$R^L = \tau^{LA} LI \tag{A.75}$$

Finally, total tax revenue is just the sum of the components discussed above, plus a lump sum tax R^{LS} and an exogenous amount of nontax revenues, R^{NT}[6]:

---

6. The lump sum tax was included to enable government revenue to be fixed in certain simulations. Ordinarily, it was zero.

$$R = R^S + R^T + R^P + R^W + R^C + R^L + R^{LS} + R^{NT} \qquad (A.76)$$

Once tax revenue is known, a number of additional adjustments produces the value of spending on goods and services. First, tax collections are supplemented by the capital income earned by government enterprises (sector 35), given by the variable KI^G. From this total, the current budget surplus (given exogenously) is deducted. Next, the interest paid to domestic and foreign holders of government bonds is subtracted. These variables are INT^{GD} and INT^{GF}, respectively. Then, domestic and foreign transfer payments, TR^{GD} and TR^{GF}, are deducted.

Finally, the government pays interest on bonds (INT^{SS}) held by Social Security and other social insurance funds. Social security can be regarded as being owned directly by the private sector, so interest payments to it can be passed on to households. Since the government does receive some income for administering the funds on behalf of the private sectors (R^{SS}), the net interest payment to households is difference of these, $INT^{SS} - R^{SS}$. All of the transactions just discussed are summarized in the equation below:

$$GE = R + KI^G - GS - (INT^{GD} + INT^{GF}) - (TR^{GD} + TR^{GF}) - (INT^{SS} - R^{SS}) \qquad (A.77)$$

where GE is government expenditure on goods and services. This is allocated to individual commodities using an exogenous vector of shares, ω^{GE}:

$$GOV_i = GE \cdot \omega_i^{GE} \qquad (A.78)$$

As described in Appendix B, the shares are determined from historical data. This completely determines the government final demand column.

A.8. Exports

For each commodity, the quantity of exports was determined from a demand equation of the form below[7]:

$$Q_i^{EXP} = A_i \left(\frac{P_i^C}{e} \right)^{\eta_i} \tag{A.79}$$

where the elasticity η is negative, and varies across commodities. Thus, exports are a function of the relative price of domestic goods to foreigners. Travel by foreigners in the United States is treated in a special way: the quantity of travel, *TRAV* is exogenous, but its allocation to commodities is endogenous. Specifically, *TRAV* is multiplied by the price of aggregate consumption, P^{CC}, and this value is divided among commodities in proportion to the split of total domestic consumption into specific goods (excluding primary factors). That is, the value of commodity i purchased by foreigners travelling the United States is:

$$TRAVEL_i = P^{CC} \cdot TRAV \frac{CON_i}{\sum_{j=1}^{35} CON_j} \tag{A.80}$$

Adding this to the value of ordinary exports produces the total value of each commodity exported:

$$EXP_i = Q_i^{EXP} \cdot P_i^C + TRAVEL_i \tag{A.81}$$

This fully determines the exports column of final demand.

7. This is similar to the treatment used in the ORANI model by Dixon, Parmenter, Sutton and Vincent (1982).

A.9. Industry and commodity output

Once the final demand vectors for consumption, investment, government spending, and exports and the share of commodities produced domestically are known, it is possible to compute industry output. To see this, let *FD* be a vector of final demands gross of imports, so element *i* is given by:

$$FD_i = CON_i + INV_i + GOV_i + EXP_i \tag{A.82}$$

If *IO* is a matrix of input-output coefficients (column shares of the interindustry use table), *IND* is a vector of industry outputs before sales taxes[8], and *COM* is a vector of commodity outputs, then the following expression must hold:

$$IO \cdot IND + FD - IMP = COM \tag{A.83}$$

This says that intermediate plus final demands for commodities equal their total output. Gross industry output can be computed from net output as shown:

$$IND^G = (I + \tau^S) \cdot IND \tag{A.84}$$

The make table gives the industry outputs (rows) composing each commodity (column). Dividing each row by its total produces a matrix of shares giving the fraction of industry output devoted to each commodity. If M^R is the matrix of row shares, then the following relationship must hold between industry and commodity output:

8. The industry outputs could be defined to be post-tax, but then the columns of the input-output table would have to be determined accordingly, which would mean that they would add up to one minus the sales tax rate, instead of one.

$$IND^{G\prime} M^{R} = COM^{\prime} \tag{A.85}$$

For convenience, define vector COM^{TS} to be the total domestic supply of each commodity, including both domestic production and imports:

$$COM^{TS} = COM + IMP \tag{A.86}$$

If W is a diagonal matrix whose element W_{ii} is the share of domestic production in the total supply of commodity i, then the value of imports must satisfy:

$$IMP = (I - W) \cdot COM^{TS} \tag{A.87}$$

Substituting out COM^{TS} and rearranging produces the following expression:

$$IMP = W^{-1} \cdot (I - W) \cdot COM \tag{A.88}$$

Substituting this into the equation for total demand for commodities and collecting terms gives the following:

$$IO \cdot IND + FD = (I + W^{-1} \cdot (I - W)) \cdot COM \tag{A.89}$$

Next, observe that since W is a diagonal matrix, the equation below must hold:

$$I + W^{-1} \cdot (I - W) = W^{-1} \tag{A.90}$$

Finally, applying this and the expressions for commodity output in terms of industry output and gross industry output in terms of net output produces the equation below:

$$(W^{-1} M^{R\prime}(I + \tau^{S}) - IO) \cdot IND = FD \tag{A.91}$$

Once FD, W, and IO are known, computing industry output is simply a matter of solving this system of equations.[9] After IND has been found, the gross output of domestic commodities can be determined as shown:

$$COM = IND \cdot (I + \tau^{S}) \cdot M^{R} \tag{A.92}$$

A.10. Imports

After gross commodity output is obtained, calculating the value of competitive imports is straightforward. For commodity i, imports are just

$$IMP_i = COM_i \cdot \frac{1 - W_{ii}}{W_{ii}} \tag{A.93}$$

Noncompeting imports (NCI) are determined differently. The input-output coefficients in the noncompeting imports row give the share of NCI in the value of industry output. When output is known, the quantity of NCI used by each industry is given by the following:

$$N_i = \frac{IND_i \cdot IO_{Ni}}{P_{i_N}} \tag{A.94}$$

9. In practice, this is done using an algorithm related to gaussian elimination.

where IO_{Ni} is the input-output coefficient for industry i in the NCI row (N). Noncompeting imports are also purchased by several final demand sectors. The quantity consumed can be found by dividing the total amount spent on NCI by the price of NCI to that sector. For consumption, this means that

$$N_C = \frac{CON_N}{P_C^N} \tag{A.95}$$

For investment, the quantity of NCI used is

$$N_I = \frac{INV_N}{P_I^N} \tag{A.96}$$

Finally, the amount of NCI purchased by the government is

$$N_G = \frac{GOV_N}{P_G^N} \tag{A.97}$$

A.11. Factor Markets

In addition to equations describing the behavior of agents, the model also has a set of expressions which define equilibrium in the factor markets.[10] For capital, the quantity demanded by each industry can be determined from industry output and the appropriate input-output coefficients:

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10. There are no explicit equations describing equilibrium in the output markets because the way prices and industry output were determined means they would be satisfied automatically. Technically, these equations have already been used elsewhere in the model.

$$K_i = \frac{IND_i \cdot IO_{K,i}}{P_i^K} \tag{A.98}$$

The quantity of capital services demanded by households is simply the amount they spend on it divided by the price:

$$K_C = \frac{CON_K}{P_C^K} \tag{A.99}$$

No other final demand sectors purchase capital services. Total capital demanded is the weighted sum of the capital services demanded by industries and households. The weights differ from 1 because of the method used to construct the original data (refer to Appendix B). Thus, the total capital demanded is:

$$K^D = \sum_{i \in DEM} K_i \cdot S_i^K \tag{A.100}$$

Finally, this must equal the total available capital stock. Unfortunately, another aggregation constant enters this equation, so capital market equilibrium requires

$$S_K \cdot K^S = K^D \tag{A.101}$$

The labor market is very similar to that for capital. The quantity demanded by industry *i* is determined by its output and the relevant input-output coefficient:

$$L_i = \frac{IND_i \cdot IO_{L,i}}{P_i^L} \tag{A.102}$$

Households and the government also demand labor:

$$L_C = \frac{CON_L}{P_C^L} \tag{A.103}$$

$$L_G = \frac{GOV_L}{P_G^L} \tag{A.104}$$

Total labor demanded is the sum of these components, again with aggregation weights appearing:

$$L^D = \sum_{i \in DEM} L_i \cdot S_i^L \tag{A.105}$$

The labor market is in equilibrium when the quantity of labor supplied by households satisfies the following expression:

$$P^H H = LI \cdot (1 - \tau^{LM}) + P^{LEIS} \cdot LEIS \tag{A.106}$$

where τ^{LM} is the marginal tax rate on labor income, H is the total number of hours available to be allocated between labor and leisure, LI is labor income, and P^H is the value households place on an hour of time.

A.12. Income, Wealth and Savings

At this point household income can be computed. Gross labor income is the sum of the payments received from all sectors that demand labor:

$$LI = \sum_{i \in DEM} L_i \cdot P_i^L \tag{A.107}$$

Similarly, gross capital income is just

$$KI = \sum_{i \in DEM} K_i \cdot P_i^K \tag{A.108}$$

Using the value of new capital goods determined in the investment model, it is possible to determine the value of the capital stock:

$$V^K = P^K K^S \tag{A.109}$$

Once this is known, solving for depreciation is straightforward:

$$D = \delta \cdot V^K \tag{A.110}$$

where δ is the depreciation rate. Capital gains is the growth of the value of the capital stock less depreciation:

$$CG = \left(\frac{\dot{P}^K}{P^K} - \delta \right) \cdot V^K \tag{A.111}$$

The capital income paid out by firms after taxes is dividends, DIV, and is just gross capital income less the capital income tax, less property taxes on the value of the capital stock:

$$DIV = KI \cdot (1 - \tau^K) - V^K \tau^P \tag{A.112}$$

Part of this, the fraction earned by government enterprises, accrues to the government, and is known as KI^G:

$$KI^G = K_{35} P_{35}^K (1 - \tau^K) \tag{A.113}$$

Using the arbitrage equation below relating the return on debt to that on equity, it is possible to determine the interest rate:

$$r \cdot V^K = DIV + CG \tag{A.114}$$

This requires the nominal interest rate on debt to be equal to the dividends and capital gains earned on equity divided by the value of the equity. Rewriting the expression and substituting in the value of several variables produces the following equation, which determines the interest rate:

$$r = \frac{DIV + CG}{V^K} = \frac{KI\,(1 - \tau^K)}{V^K} + \frac{\dot{P}^K}{P^K} - \delta - \tau^P \tag{A.115}$$

From the point of view of households, government and foreign debt pays the interest rate in every period, so interest income is

$$INT = r\,(DEBT^G + DEBT^R) \tag{A.116}$$

where $DEBT^G$ and $DEBT^R$ are government and foreign debt held by domestic households. As discussed in Appendix B, during the sample period actual interest payments differed somewhat from the values implied by this formula. To improve the model's behavior, the actual government and foreign interest payments were exogenized, and two adjustment variables were computed to balance what households receive with interest actually paid. For the government, this adjustment was IA^G, and could be computed using the expression below:

$$IA^G = INT^{GD} - r \cdot DEBT^G + (INT^{SS} - R^{SS}) \tag{A.117}$$

For foreign debt, the adjustment was just:

$$IA^R = INT^{RD} - r \cdot DEBT^R \tag{A.118}$$

These two adjustments were then added to household income.

Since both types of bonds pay the nominal rate of interest in every period, the market value of the debt is just its face value. This means that the total financial wealth of households is just the sum of the value of debt and the value of capital:

$$FW = V^K + DEBT^G + DEBT^R \tag{A.119}$$

where FW is total financial wealth. Post-tax household income is just post-tax labor income plus dividends net of government capital income plus interest less wealth taxes plus transfers from the government, the lump sum tax and the two interest adjustments describe above:

$$Y = LI\,(1 - \tau^{LA}) + DIV - KI^G + INT \tag{A.120}$$

$$- FW\,\tau^W + IA^G + IA^R + TR^{GD} + R^{LS} \tag{A.121}$$

Savings is income less consumption expenditure less personal transfers to foreigners less nontax payments:

$$S = Y - CE - TR^{PF} - R^{NT} \tag{A.122}$$

Finally, balance in the financial market requires that private plus government savings equal investment plus the current account surplus:

$$S + GS = CA + INV \tag{A.123}$$

This expression can be regarded as determining the value of investment, since the government surplus is exogenous, and the current account balance is determined by a number of factors, as discussed in the section below.

A.13. Foreign Sector Accounts

To compute the current account surplus, the first step is to add up the foreign value (before tariffs) of all imports. For noncompeting imports, this is given by the expression below:

$$V^{NCI} = \sum_{i \in DEM} NCI^i \cdot P_i^{NF} \tag{A.124}$$

where P_i^{NF} is the foreign price (F) of noncompeting imports (N) of commodity i. For ordinary imports, the total foreign value is

$$V^{IMP} = \sum_{i=1}^{35} \frac{IMP_i}{1 + \tau_i^T} \tag{A.125}$$

Since *IMP* is the price of imports to domestic buyers, it includes the tariff. Thus, to obtain the foreign value it is necessary to divide by one plus the tariff rate. The foreign value of exports is easier to compute. Since there are no export subsidies in the model, the foreign value of exports is exactly equal to their domestic value:

$$V^{EXP} = \sum_{i=1}^{35} EXP_i \tag{A.126}$$

The current account surplus is the value of exports less the value of imports plus interest received on domestic holdings of foreign bonds less private and government transfers abroad, less interest on government bonds paid to foreigners. In algebra, the current account surplus is the following:

$$CA = V^{EXP} - V^{IMP} - V^{NCI} + INT^{RD} - (TR^{PF} + TR^{GF}) - INT^{GF} \tag{A.127}$$

where INT^{RD} is the interest received on foreign debt, TR^{PF} and TR^{GF} are private and government transfers, and INT^{GF} are government interest payments abroad.

This completes the specification of the intraperiod model. Given values of the capital stock, number of hours and other state and costate variables in a particular year, the equations above can be solved for that year's equilibrium prices and quantities. The next section describes the accumulation conditions that form the fundamental relationship between one period and its successor.

A.14. Intertemporal Equations

Several state variables in the model accumulate endogenously. The most obvious is the capital stock, which is supplemented by net investment every year. The differential equation below determines the change in the capital stock from one year to the next:

Capital accumulation:

$$\dot{K}^S = S^I \cdot \frac{INV}{P^{II}} - \delta \cdot K^S \tag{A.128}$$

The term S^I is an aggregation constant (see Appendix B). The stock of government debt held domestically is subject to a similar condition. Its growth is mostly determined by the government surplus, but several small adjustments also contribute. Specifically, the change in government debt is government foreign investment, GOV^{FI} less the government surplus plus two discrepancies appearing in the sample period data: the government debt discrepancy $DISC^{GD}$ and capital gains on government debt, CG^{GD}:

$$DEBT^G = GOV^{FI} - GS + DISC^{GD} + CG^{GD} \tag{A.129}$$

The evolution of government debt held by foreigners is somewhat simpler, depending only on government foreign investment and capital gains on government debt held by foreigners CG^{GDF}:

$$DEBT^{GF} = -\left(GOV^{FI} + CG^{GDF}\right) \tag{A.130}$$

Finally, the accumulation of foreign assets by the domestic sector depends on the current account surplus less government foreign investment plus debt and capital gain discrepancies:

$$DEBT^R = CA - GOV^{FI} + DISC^{RD} + CG^{RD} \tag{A.131}$$

where $DISC^{RD}$ is the foreign debt discrepancy, and CG^{RD} is capital gains on foreign debt.

A.15. Other Equations

Several additional equations were included in the model for convenience. All were essentially definitional, simply computing certain conventional aggregates. The first of these was labor-deflated gross national product, GNP^N:

$$GNP^N = CE + INV + GE + V^{EXP} + INT^{RD} + R^T - V^{IMP} - V^{NCI} - INT^{GF} \qquad \text{(A.132)}$$

From this, real GNP could be computed by dividing by the price of consumption goods:

$$GNP^R = \frac{GNP^N}{P^{CC}} \qquad \text{(A.133)}$$