Appendix A.

Equations of the Model & Glossary

A.0 Notation:

Time
t ε I_T $I_T = \{1, 2, \ldots T, \ldots\}$

Industry/Producer
j ε I_{IND} $I_{IND} = \{1, 2, \ldots, 35\}$

IO Commodities
i ε I_{COM} $I_{COM} = \{1, 2, \ldots, 35\}$

Industry Inputs
i ε I_{INP} $I_{INP} = \{1, 2, \ldots, 35, NCI, K, L\}$

NIPA PCE Commodities
n ε I_{PCE} $I_{PCE} = \{1, 2, \ldots, 38\}$

Purchasers of domestic output
j ε I_{BUY} $I_{BUY} = \{1, 2, \ldots, 35, C, I, G, X\}$

Households
k ε I_{POP}

Nodes of production function
m ε I_{PNODE} $I_{PNODE} = \{EN, M, \ldots\ldots WP\}$
i ε I_{PNODEm} I_{PNODEm} in Table ?.?

Nodes of consumption function
m ε I_{CNODE} $I_{CNODE} = \{EN, FD, \ldots\ldots RS\}$
i ε I_{CNODEm} I_{CNODEm} in Table ?.?

Nodes of investment function
m ε I_{INV} $I_{INV} = \{fixed, \ldots, mining\}$
i ε I_{INVm} I_{INVm} in Table ?.?

Vector of 1’s
ι

Transpose of matrix A
A’

Diagonal matrix of a vector v
$Diag(v)$

A.1 Household Sector

Household first stage decision; Euler equation.

$$\text{Max} \sum_{t=1}^{\infty} \frac{N_t^{eq}}{(1+\rho)^t} (F_t/N_t^{eq})^{1-\frac{1}{\sigma}} \tag{A.1.1}$$

subject to

$$WF \equiv PK_0K_0 + BG_0 + BF_0 + \sum_{t=1}^{\infty} \frac{\bar{w}_t LH_t + misc_t}{\prod_{s=1}^{t} 1+r_s} \geq \sum_{t=1}^{\infty} \frac{PF_t F_t}{\prod_{s=1}^{t} 1+r_s} \tag{A.1.2}$$

where

$$misc = GTRAN + (1-tk)(GINT^{rec} + YROW^{rec}) - twW_{t-1} - TLUMP - CR - TAXN$$

$$\left[\frac{F_t/N_t^{eq}}{F_{t-1}/N_{t-1}^{eq}} \right]^{\frac{1}{\sigma}} = \frac{1+r_t}{1+\rho} \frac{PF_{t-1}}{PF_t} \tag{A.1.3}$$

Household second stage decision, goods-leisure choice.

$F_t = F(CC_t, LEIS_t)$ Formulated with the price dual: (A.1.4)

$$VV(PCC, w^{LE}, MF) = \max \; F(CC, LEIS) \qquad s.t. \;\; MF = PCC.CC + w^{LE}.LEIS$$

$$-\ln VV = \alpha^{F\prime} \ln \frac{PF^F}{MF} + \tfrac{1}{2} \ln \frac{PF^{F\prime}}{MF} B^F \ln \frac{PF^F}{MF} \tag{A.1.5}$$

$$= \alpha^{F\prime} \ln PF^F + \tfrac{1}{2} \ln PF^{F\prime} B^F \ln PF^F - \ln MF$$

$$\ln PF^F \equiv (PCC, w^{LE})'$$

$$\ln PF = \alpha^F \ln PF^F + \tfrac{1}{2} \ln PF^{F\prime} B^F \ln PF^F \tag{A.1.6}$$

$$= (\alpha_0^F + \frac{\beta_0^F}{1+\exp(-\mu^F(t-\tau^F)}) \ln PF^F + \tfrac{1}{2} \ln PF^{F\prime} B^F \ln PF^F$$

$$SF \equiv \begin{bmatrix} PCC.CC/MF \\ w^{LE}.LEIS/MF \end{bmatrix} = \alpha^F + B^F \ln PF^F \tag{A.1.7}$$

$$CC = SF_1 * MF/PCC \tag{A.1.8}$$

$$MF = PF.F \tag{A.1.9}$$

$$= PCC.CC + w^{LE}.LEIS \tag{A.10}$$

$$LS = LH - \psi_C^L LEIS \tag{A.1.11}$$

$$\bar{w}LH = (1 - tl^a)\frac{wLS}{1 - tl^m} + w\psi_C^{LE} LEIS$$

Income and savings:

$$W_t \equiv PK_t K_t + BG_t + BF_t \tag{A.1.12}$$

$$\begin{aligned} YF_t = {} & (1 - tk)\,(PKD_t KD_t - PKD_{35} KD_{35}) - tpPK_{t-1} K_{t-1} + r_t BF_{t-1} \\ & + (1 - tk) YROW_t^{rec} + r_t BG_{t-1} + (1 - tk) GINT_t^{rec} + \bar{w}_t LH_t \\ & + GTRAN_t - TLUMP_t - twW_{t-1} \end{aligned} \tag{A.1.13}$$

$$\begin{aligned} Y_t = {} & (1 - tk)\,(PKD_t KD_t - PKD_{35} KD_{35}) - tpPK_{t-1} K_{t-1} + r_t BF_{t-1} \\ & - (1 - tk) YROW_t^{rec} + r_t BG_{t-1} - (1 - tk) GINT_t^{rec} + w_t LS_t \frac{1 - tl^a}{1 - tl^m} \\ & + GTRAN_t - TLUMP_t - twW_{t-1} \end{aligned} \tag{A.1.14}$$

$$\begin{aligned} S_t &= YF_t - PF_t F_t - CR_t - TAXN_t \\ &= YF_t - PCC_t CC_t - w_t LH_t - CR_t - TAXN_t \\ &= Y_t - PCC_t CC_t - CR_t - TAXN_t \end{aligned} \tag{A.1.15}$$

Household third stage decision, allocation of consumption goods:

NESTED STRUCTURE OF CONSUMPTION (A.1.16)

$CC = CC(N^{EN}, N^{FD}, N^{ND}, N_K, N^{SV})$	Commodity aggregate
$N^{EN} = N^{EN}(N_6, N^{FL}, N_{18}, N_{19})$	Energy
$N^{FD} = N^{FD}(N_1, N_2, N_3, N_9)$	Food agg.
$N^{ND} = N^{ND}(N^{CS}, N^{HA}, N_{12}, N^{NM})$	Non-durables
$N^{SV} = N^{SV}(N^{HR}, N^{HS}, N^{TR}, N^{MD}, N^{SM})$	Services agg.
$N^{FL} = N^{FL}(N_7, N_8)$	Fuel and wood
$N^{CS} = N^{CS}(N_4, N_5)$	Clothing and shoes
$N^{HA} = N^{HA}(N_{10}, N_{11})$	Household articles
$N^{NM} = N^{NM}(N_{13}, N_{14}, N_{15}, N_{16})$	Miscellaneous non-durables

$N^{HR} = N^{HR}(N_{17}, N_{34})$ Rental housing

$N^{HS} = N^{HS}(N_{20}, N_{21}, N_{22}, N_{23})$ Household services

$N^{TR} = N^{TR}(N_{24}, N_{25})$ Transportation

$N^{MD} = N^{MD}(N_{26}, N_{27})$ Medical

$N^{SM} = N^{SM}(N_{28}, N^{BS}, N^{RS}, N_{32})$ Miscellaneous services

$N^{BS} = N^{BS}(N_{29}, N_{30})$ Business services

$N^{RS} = N^{RS}(N_{31}, N_{33})$ Recreation

subscripts $\varepsilon\ I_{PCE}$

Dual of **top** (m=1) tier consumption demands CC=CC(...) :

$$\ln V_k = \alpha^{H1} \ln P^{H1} + \tfrac{1}{2} \ln P^{H1}\ B^H \ln P^{H1} + (1 - \ln P^{H1} B\iota) \ln M_k + \ln P^{H1} B_{pA} A_k \qquad \text{(A.1.17)}$$

$$\ln P^{H1} = (\ln PN^{EN}, \ln PN^{FD}, PN^{ND}, PN^{K}, PN^{SV}) \qquad k\ \varepsilon\ I_{POP}$$

$$\ln PCC = \iota' B^H \ln P^{H1} \ln \frac{PCC.CC}{N^{eq}} - \alpha^H \ln P^{H1} + \tfrac{1}{2} \ln P^{H1} B^H \ln P^{H1} \qquad \text{(A.1.18)}$$

$$SN^{TOP} \equiv \left(\frac{PN^{EN} N^{EN}}{PCC.CC}, \frac{PN^{FD} N^{FD}}{PCC.CC}, \frac{PN^{ND} N^{ND}}{PCC.CC}, \frac{PN^{K} N^{K}}{PCC.CC}, \frac{PN^{SV} N^{SV}}{PCC.CC} \right)' \qquad \text{(A.1.19)}$$

$$= \frac{\alpha^H + B^H \ln P^{H1} - B^H \iota\, \xi^d + B_{pA}\, \xi^L}{-1 + \iota' B^H \ln P^{H1}}$$

Price dual of **lower** tiers consumption demands $N^m(\ldots)$:

$$\ln PN^m = \alpha^{Hm\prime} \ln P^{Hm} + \tfrac{1}{2} \ln P^{Hm\prime} B^{Hm} \ln P^{Hm} \qquad m\ \varepsilon\ I_{CNODE} \qquad \text{(A.1.20)}$$

$$\ln P^{Hm} \equiv (\ln PN_{m1}, \ldots, \ln PN_{mi}, \ldots, \ln PN_{m,im})' \qquad i\ \varepsilon\ I_{CNODEm} \qquad \text{(A.1.21)}$$

$$SN^m = \begin{bmatrix} PN_{m1} N_{m1} / PN^m N^m \\ \cdots \\ PN_{m,im} N_{m,im} / PN^m N^m \end{bmatrix} = \alpha^{Hm} + B^{Hm} \ln PN^{Hm} \quad \begin{matrix} m\ \varepsilon\ I_{CNODE} \\ mi\ \varepsilon\ I_{CNODEm} \end{matrix} \qquad \text{(A.1.22)}$$

$PN_{mi}\ \varepsilon\ \{PN_1, \ldots, PN_{34,}\ PN^{EN}, \ldots, PN^{RS}\}$

$N_{mi}\ \varepsilon\ \{N_1, \ldots, N_{34,}\ N^{EN}, \ldots, N^{RS}\}$

$$PN_1N_1 = SN_1^{FD} * SN_2^{TOP} * PCC.CC \qquad \text{(A.1.23)}$$

$$PN_2N_2 = SN_2^{FD} * SN_2^{TOP} * PCC.CC$$

. . . . . .

$$PN_{34}N_{34} = SN_2^{HR} * SN_1^{SV} * SN_5^{TOP} * PCC.CC$$

$$VN \equiv (\, PN_1N_1, \ldots, PN_{34}N_{34}, PKD_CKD_C \,)'$$

$$PN = \mathbf{H}'\, PS^H \qquad \text{(A.1.24)}$$

where $PS^H = (PS_1, \ldots, PS_{35}, PNCI_C, PKD_C, PLD_C)$.

$$VC \equiv (\, PS_1C_1, \ldots, PS_{35}C_{35}, \ldots, PLD_CLD_C \,)' \qquad \text{(A.1.25)}$$

$$= \mathbf{H}\, VN$$

$$C_i = VC_i/PS_i \qquad i\ \varepsilon\ I_{INP} \qquad \text{(A.1.26)}$$

$$C^P \equiv (C_1, C_2, \ldots, C_{35})'$$

$$C \equiv (C_1, \ldots, C_{35}, NCI_C, KD_C, LD_C)'$$

A.2 Producer model

NESTED STRUCTURE OF PRODUCTION (A.2.1)

$QI_j = QI^j(KD_j, LD_j, QP^{jE}, QP^{jM})$ Industry output

$QP^{jE} = QP^E(QP^j_3, QP^j_4, QP^j_{16}, QP^j_{30}, QP^j_{31})$ Energy aggregate
$QP^{jM} = QP^M(QP^j_6, QP^{jMA}, QP^{jME}, QP^{jMN}, QP^{jMS})$ Material aggregate

$QP^{jMA} = QP^{AG}(QP^j_1, QP^j_7, QP^j_8, QP^{jTX}, QP^{jWP})$ Agriculture products
$QP^{jME} = QP^{ME}(QP^{jFM}, QP^{jMC}, QP^{jEQ})$ Metallic agg.
$QP^{jMN} = QP^{MN}(QP^j_5, QP^j_{15}, QP^j_{17}, QP^j_{19}, QP^j_{27})$ Non-metallic agg.
$QP^{jMS} = QP^{MS}(QP^j_{28}, QP^j_{32}, QP^j_{33}, QP^j_{34}, QP^{jOS})$ Service agg.

$QP^{jTX} = QP^{TX}(QP^j_9, QP^j_{10}, QP^j_{18})$ Textile agg.
$QP^{jWP} = QP^{WP}(QP^j_{11}, QP^j_{12}, QP^j_{13}, QP^j_{14})$ Wood and paper agg.
$QP^{jFM} = QP^{FM}(QP^j_2, QP^j_{20}, QP^j_{21})$ Primary metal agg.
$QP^{jMC} = QP^{MC}(QP^j_{22}, QP^j_{23})$ Machinery agg.
$QP^{jEQ} = QP^{EQ}(QP^j_{24}, QP^j_{25}, QP^j_{26})$ Equipment agg.
$QP^{jOS} = QP^{OS}(QP^j_{29}, QP^j_{35}, QP^j_{NCI})$ Miscellaneous services

Price dual of **top** tier of production functions $QI_j = QI(\dots)$:

$PO_j = PO^j(PKD_j, PLD_j, PP^{jE}, PP^{jM}) \qquad j \varepsilon I_{IND}$

$$\ln PO_j = \alpha^j_0 + \alpha^{Pj\prime} \ln P^{Pj0} + \tfrac{1}{2} \ln P^{Pj0\prime} B^{Pj} \ln P^{Pj0} + \ln P^{Pj0\prime} B^j_{pt} g(t) + \alpha^j_t\, g(t) + \tfrac{1}{2}\, \beta^j_{tt} g(t)^2 \qquad \text{(A.2.2)}$$

where $g(t) = \dfrac{1}{1+\exp(-\mu^j(t-\tau^j))}$.

and $\ln P^{Pj0} \equiv (\ln PKD_j, \ln PLD_j, \ln PP^{jEN}, \ln PP^{jM})'$

$$SP^{jTOP} = \begin{bmatrix} PKD_j KD_j / PO_j QI_j \\ .. \\ PP^{jM} QP^{jM} / PO_j QI_j \end{bmatrix} = \alpha^{Pj} + B^{Pj} \ln P^{Pj0} + B^j_{pt}\, g(t) \qquad \text{(A.2.3)}$$

Price dual of **lower** tiers of production functions, $QP^{jm} = QP(\dots)$:

$$\ln PP^{jm} = \alpha^{jm}_0 + \alpha^{Pjm\prime} \ln P^{Pjm} + \tfrac{1}{2} \ln P^{Pjm\prime} B^{Pjm} \ln P^{Pjm} \qquad m \varepsilon I_{PNODE} \qquad \text{(A.2.4)}$$

$$\ln P^{Pjm} \equiv (\ln PP^{j}_{m1}, \ldots, \ln PP^{j}_{mi}, \ldots, \ln PP^{j}_{m,im})' \qquad i \varepsilon I_{PNODEm}$$

$$SP^{jm} = \begin{bmatrix} PP^{j}_{m1} QP^{j}_{m1} / PP^{jm} QP^{jm} \\ .. \\ PP^{j}_{m,im} QP^{j}_{m,im} / PP^{jm} QP^{jm} \end{bmatrix} = \alpha^{Pjm} + B^{Pjm} \ln P^{Pjm} \qquad \text{(A.2.5)}$$

$$PP^{j}_{mi} \; \varepsilon \; \{PS_1, \ldots, PS_{35}, PNCI_j, PP^{jMA}, \ldots, PP^{jOS}\}$$

$$QP^{j}_{mi} \; \varepsilon \; \{QP^{j}_{1}, \ldots, QP^{j}_{35}, NCI_j, QP^{jMA}, \ldots, QP^{jOS}\}$$

$$VP^{j} \equiv (\, PS_1 QP^{j}_{1}, \ldots PS_{35} QP^{j}_{35}, PNCI_j NCI_j, PKD_j KD_j, PLD_j LD_j \,)'$$

$$VQI \equiv (\, PO_1 QI_1, \ldots, PO_{35} QI_{35} \,)'$$

Taxes.

$$PI_j = (1 + tt_j) PO_j \qquad j \; \varepsilon \; I_{IND} \qquad \text{(A.2.6)}$$

$$VQI^{tt} \equiv (\, PI_1 QI_1, \ldots, PI_{35} QI_{35} \,)'$$

$$= Diag(\iota + tt)\, VQI$$

Commodities from industry outputs:

$$PC = \mathbf{M}'\, PI \qquad \text{(A.2.7)}$$

$$VQC \equiv (\, PC_1 QC_1, \ldots, PC_{35} QC_{35} \,)' \qquad \text{(A.2.8)}$$

$$= \mathbf{M}'\, VQI^{tt}$$

$$QC_i = VQC_i / PC_i \qquad i \; \varepsilon \; I_{COM} \qquad \text{(A.2.9)}$$

The input-output USE matrix:

$$A_{1j} = SP^{jAG}_{1} * SP^{jM}_{2} * SP^{jTOP}_{4} \qquad \text{(A.2.10)}$$

$$A_{2j} = SP^{jFM}_{1} * SP^{jME}_{1} * SP^{jM}_{3} * SP^{jTOP}_{4}$$

. . . . . .

$$A_{35j} = SP^{jOS}_{2} * SP^{jMS}_{5} * SP^{jM}_{5} * SP^{jTOP}_{4}$$

$$A_j \equiv (A_{1j}, A_{2j}, \ldots, A_{35j})' \qquad j \varepsilon I_{IND} \tag{A.2.11}$$

$$\mathbf{A} = [A_1, A_2, \ldots, A_{35}] \tag{A.2.12}$$

$$PNCI_j NCI_j = SP_3^{jOS} * SP_5^{jMS} * SP_5^{jM} * SP_4^{jTOP} * PO_j * QI_j \tag{A.2.13}$$

$$PKD_j KD_j = SP_1^{jTOP} * PO_j * QI_j \tag{A.2.14}$$

$$PLD_j LD_j = SP_2^{jTOP} * PO_j * QI_j \tag{A.2.15}$$

$$KD_4 = \overline{KD}_4 \qquad \text{(oil sector)} \tag{A.2.16}$$

A.3 The Bank and Investment demands

$$\text{Max} \sum_{t=u}^{\infty} \frac{(1-tk)(PKD_t \psi^K K_{t-1} - tpPK_{t-1}K_{t-1}) - PII_t I_t^a}{\prod_{s=u}^{t} 1+r_s} \tag{A.3.1}$$

subject to

$$K_t = (1-\delta)K_{t-1} + \psi^I I_t^a \tag{A.3.2}$$

Hamiltonian :

$$\frac{(1-tk)(PKD_t \psi^K K_{t-1} - tpPK_{t-1}K_{t-1}) - PII_t I_t^a}{\prod_{s=u}^{t} 1+r_s} + \frac{\lambda_t}{\prod_{s=u}^{t} 1+r_s}((1-\delta)K_{t-1} + \psi^I I_t^a - K_t) \tag{A.3.3}$$

Euler equation :

$$(1+r_t)\frac{PII_{t-1}}{\psi_{t-1}^I} = (1-tk)(PKD_t \psi_t^K - tp\ PK_{t-1}) + (1-\delta)\frac{PII_t}{\psi_t^I} \tag{A.3.4}$$

Aggregation relationships.

$$PK_t = \psi_t^{PK}\ PII_t \tag{A.3.5}$$

$$KD_t = \psi_t^K\ K_{t-1} \tag{A.3.6}$$

$$\psi_t^I = \alpha^I + \frac{\beta^I}{1+\exp(-\mu^I(t-\tau^I))} \tag{A.3.7}$$

$$\psi_t^{PK} = \alpha^{PK} + \frac{\beta^{PK}}{1+\exp(-\mu^{PK}(t-\tau^{PK}))} \tag{A.3.8}$$

$$VII = PII.\ I^a \tag{A.3.9}$$

NESTED STRUCTURE OF INVESTMENT (A.3.10)

$I^a = I^a(I^{fixed}, I^{inventory})$ Aggregate investment

$I^{fixed} = I^{FX}(I^{long}, I^{short})$ Fixed investment agg.
$I^{inventory} = I^{IY}$ Change in business inventories
$I^{long} = I^{LG}(I_6, I_{33})$ Long-lived investment agg.
$I^{short} = I^{SH}(I^{vehicles}, I^{machinery}, I^{services})$ Short-lived investment agg.

$I^{vehicles} = I^{VE}(I_{24}, I_{25})$ Vehicle agg.
$I^{machinery} = I^{MC}(I_{22}, I_{23}, I^{other-m})$ Machinery agg.
$I^{services} = I^{SV}(I_{32}, I^{other-s})$ Services agg.

$I^{other-m} = I^{MO}(I^{gadgets}, I^{wood}, I^{nonmetal}, I^{misc})$ Other machinery agg.
$I^{other-s} = I^{SO}(I_{34}, I^{movers})$ Other services agg.

$I^{gadgets} = I^{GD}(I_{20}, I_{21}, I_{26})$ Metals and instruments agg.
$I^{wood} = I^{WD}(I_{11}, I_{12})$ Wood products agg.
$I^{nonmetal} = I^{MN}(I_{15}, I_{17}, I_{19}, I_{27})$ Nonmetallic products agg.
$I^{misc} = I^{OO}(I^{textile}, I_{13}, I^{mining})$ Miscellaneous agg.
$I^{mover} = I^{TC}(I_{28}, I_{29})$ Transportation and Communications agg.

$I^{textile} = I^{TX}(I_9, I_{10}, I_{18}, I_{NCI})$ Textile agg.
$I^{mining} = I^{MG}(I_2, I_4)$ Minerals agg.

At top tier of investment functions I=I(...) :

$$VII = VII^{fixed} + VII^{invy} \tag{A.3.11}$$

$$\frac{VII^{invy}}{VII} = \alpha^{IY} \tag{A.3.12}$$

Price dual of investment demand tiers $I^m = I^m(\dots)$:

$$VI_i^{invy} = \alpha_i^{IY}\, VII^{invy} \qquad i\varepsilon I_{COM} \tag{A.3.13}$$

$$\ln PII^m = \alpha^{\mathrm{Im}} \ln P^{\mathrm{Im}} + \tfrac{1}{2} \ln P^{\mathrm{Im}\prime} B^{\mathrm{Im}} \ln P^{\mathrm{Im}} \qquad m\varepsilon I_{INV} \tag{A.3.14}$$

$$\ln P^{\mathrm{Im}} \equiv (\ln PII_{m1}, \dots, \ln PII_{mi}, \dots, \ln PII_{m,im})' \qquad i\varepsilon I_{INVm}$$

$$SI^m = \begin{bmatrix} PII_{m1} I^f_{m1} / PII^m I^m \\ \cdots \\ PII_{m,im} I^f_{m,im} / PII^m I^m \end{bmatrix} = \alpha^{\mathrm{Im}} + B^{\mathrm{Im}} \ln PII^{\mathrm{Im}} \qquad \begin{matrix} m \ \varepsilon \ I_{INV} \\ mi \ \varepsilon \ I_{INVm} \end{matrix} \qquad \text{(A.3.15)}$$

$$PII_{mi} \ \varepsilon \ \{PS_1, \ldots, PS_{35}, PII^{\,fixed}, \ldots, PII^{\,mining}\}$$

$$I_{mi} \ \varepsilon \ \{I^f_1, \ldots, I^f_{35}, I^{\,fixed}, \ldots, I^{\,mining}\}$$

Values of individual commodities making up aggregate investment demand:

$$VI_1 = 0 + VI_1^{invy} \qquad \text{(A.3.16)}$$

$$VI_2 = SI_1^{MG} * SI_3^{OO} * SI_4^{MO} * SI_3^{MC} * SI_2^{SH} * SI_2^{FX} \ VII^{\,fixed} + VI_2^{invy}$$

. . . . . .

$$VI_{34} = SI_1^{SO} * SI_2^{SV} * SI_3^{SH} * SI_2^{FX} \ VII^{\,fixed} + VI_{34}^{invy}$$

$$VI_{35} = 0$$

$$I_i = VI_i / PS_i \qquad \text{(A.3.17)}$$

$$VI \equiv (\, PS_1 I_1, \ldots, PS_{35} I_{35}, PNCI_I \, NCI_I \,)'$$

$$I^P \equiv (\, I_1, \ldots, I_{35} \,)'$$

$$I \equiv (\, I_1, \ldots, I_{35}, NCI_I \,)'$$

A.4 The Government

Stock-flow relations.

$$BG_t = BG_{t-1} + \Delta G + GFI + \Delta P_t^{BG} + BG^{disc} \tag{A.4.1}$$

$$BG_t^* = BG_{t-1}^* - GFI - \Delta P_t^{BG*} \tag{A.4.2}$$

Government expenditures on goods.

$$\begin{aligned} VGG = \Delta G + \sum_j tt_j PO_j QI_j + \sum_i tr_i PM_i M_i + tkPKD.KD + tk(rBG + rBF)/(1-tk) \\ + tk(GINT^{rec} + YROW^{rec}) + tl^a wLS/(1-tl^m) + tpPK.K \\ + tw(PK.K + BG + BF) + TAXN + (1-tk)PKD_{35}KD_{35} + TAXSS \\ + TLUMP \\ - r\frac{BG}{1-tk} - GINT^{rec} - GINTR - GTRAN - GR \end{aligned} \tag{A.4.3}$$

$$VG_i = \alpha_i^G VGG \qquad i \varepsilon I_{INP} \tag{A.4.4}$$

$$G_i = VG_i / PS_i \tag{A.4.5}$$

$$VG \equiv (PS_1 G_1, \ldots, PS_{35} G_{35})'$$

$$G^P \equiv (G_1, \ldots, G_{35})'$$

$$G \equiv (G_1, \ldots, G_{35,} NCI_{G,} KD_G, LD_G)'$$

$$GFI = -GINTR - GR - GM \tag{A.4.6}$$

$$GINT^{rec} = GINT - \frac{r}{1-tk} BG_{t-1} \tag{A.4.7}$$

A.5 The Rest-of-the-World

Non-competitive imports.

$$PNCI_j = e\,(1 + tr_j^n)PNCI_j^* \qquad j = 1, \ldots, 35, C, I, G \qquad \text{(A.5.1)}$$

$$PM_i = e\,(1 + tr_j)PM_i^* \qquad i \varepsilon I_{COM} \qquad \text{(A.5.2)}$$

Competitive imports and domestic output making up total supply:

$$QS_i = QS(\,QC_i, M_i) \qquad i \varepsilon I_{COM} \qquad \text{(A.5.3)}$$

with the price dual:

$$\ln PS_i = (\alpha_0^{Mi} + \frac{\beta_0^{Mi}}{1 + e^{-\mu(t-\tau)}}) \ln P^{Mi} + \tfrac{1}{2} \ln P^{Mi\prime} B^{Mi} \ln P^{Mi} \qquad \text{(A.5.4)}$$

$$\ln P^{Mi} \equiv (\ln PC_i, \ln PM_i)'$$

$$PM_i = e\,(1 + tr_i)PM_i\,* \qquad i \varepsilon I_{COM} \qquad \text{(A.5.5)}$$

$$SD^i \equiv \begin{bmatrix} PC_i QC_i / PS_i QS_i \\ PM_i M_i / PS_i QS_i \end{bmatrix} = \alpha_0^{Mi} + \frac{\beta_0^{Mi}}{1 + \exp(-\mu^{Mi}(t - \tau^{Mi})} + B^{Mi} \ln P^{Mi} \qquad \text{(A.5.6)}$$

$$PS_i QS_i = PC_i QC_i + PM_i M_i \qquad i \varepsilon I_{COM} \qquad \text{(A.5.7)}$$

$$VQS \equiv (\,PS_1 QS_1, \ldots, PS_{35} QS_{35}\,)'$$

$$VM \equiv (\,PM_1 M_1, \ldots, PM_{35} M_{35}\,)'$$

$$SM \equiv (\,SD_2^1, SD_2^2, \ldots, SD_2^{35}\,)'$$

$$M \equiv (\,M_1, M_2, \ldots, M_{35}\,)'$$

Exports.

$$X_i = X_i(Y^*, (1 + tr_i^*)PC_i / eP_i^*) \qquad i \varepsilon I_{COM} \qquad \text{(A.5.8)}$$

$$= EX_{i0}(Y^*) \left(\frac{(1 + tr_i^*)PC_i}{e} \right)^{\eta_i} + X_i^{tr} \qquad \text{(A.5.9)}$$

$$X_i^{tr} = \frac{PCC.\,XR}{\sum PC_i C_i} C_i \qquad \text{(A.5.10)}$$

$$EX_{it} = \alpha_i^x + \lambda_i^x \ln Y_t^* \qquad \text{(A.5.11)}$$

$$X \equiv (X_1, \ldots, X_{35})'$$

$$VX \equiv (PC_1 X_1, \ldots, PC_{35} X_{35})' \qquad \text{(A.5.12)}$$

Current account and net foreign assets.

$$CA = \sum_i PC_i X_i + \frac{r}{1-tk} BF + YROW^{rec} - \sum_i PM_i M_i - \sum_j PNCI_j NCI_j \qquad \text{(A.5.13)}$$

$$- \bar{i} BG^* - GR - CR$$

$$BF_t = BF_{t-1} + CA_t - GFI + BF^{disc} + \Delta P^{BF} \qquad \text{(A.5.14)}$$

$$YROW^{rec} = YROW - \frac{r}{1-tk} BF \qquad \text{(A.5.15)}$$

A.6 Markets

Final demands.

$$VFD_i = PS_i\,(C_i^P + I_i^P + G_i^P) + PC_i\,X_i \qquad i \varepsilon I_{COM} \qquad \text{(A.6.1)}$$

$$VFD \equiv (VFD_1, \ldots, VFD_{35})'$$

$$= VC + VI + VG + VX$$

Supply equal demand for commodities.

$$PS_i QS_i = \sum_{j=1}^{35} PS_i QP_i^j + VFD_i \qquad \text{(A.6.2)}$$

$$VQS = \mathbf{A}\,VQI + VFD \qquad \text{(A.6.3)}$$

$$VQC = Diag(SM)\,VQS \qquad VQS = Diag(1/SM)\,VQC \qquad \text{(A.6.4)}$$

$$Diag(1/SM)\,VQC - \mathbf{A}\,VQI = VFD$$

$$Diag(1/SM)\mathbf{M}'\,Diag(\iota + tt)\,VQI - \mathbf{A}\,VQI = VFD$$

$$[Diag(1/SM)\mathbf{M}'\,Diag(\iota + tt) - \mathbf{A}\,]\,VQI = VFD \qquad \text{(A.6.5)}$$

Saving-investment balance.

$$VII = S - (BG_t - BG_{t-1}) - (BF_t - BF_{t-1}) \qquad \text{(A.6.6)}$$

Demand equal supply of capital.

$$PKD_j = \psi_j^K\,PKD \qquad j \varepsilon I_{BUY} \qquad \text{(A.6.7)}$$

$$\sum_{j=1}^{C} PKD_j KD_j = PKD.\,KD \qquad \text{(A.6.8)}$$

$$\sum_{j=1}^{C} \psi_j^K KD_j = KD = \psi^K K_{t-1} \qquad \text{(A.6.9)}$$

Demand equal supply of labor.

$$PLD_j = \psi_j^L\,\frac{w}{(1 - tl^m)} \qquad j\ \varepsilon I_{BUY} \qquad \text{(A.6.10)}$$

$$w^{LE} = \psi_C^L\,w \qquad \text{(A.6.11)}$$

$$(1 - tl^m) \sum PLD_j LD_j = w\,LS = w\,(LH - \psi_C^L LEIS) \qquad \text{(A.6.12)}$$

$$\sum_{j=1}^{G} \psi_j^L LD_j = LS \qquad \text{(A.6.13)}$$

A.7 Steady state equilibrium

$$Prices_T = Prices_{T-1} \quad \text{(A.7.1)}$$

$$Quantities_T = Quantities_{T-1} \quad \text{(A.7.2)}$$

$$\Delta G_T = 0 \quad \text{(A.7.3)}$$

$$CA_T = 0 \quad \text{(A.7.4)}$$

$$r_t = \rho \quad \text{(A.7.5)}$$

$$\psi^I I_T^a = \delta K^T \quad \text{(A.7.6)}$$

A.8 GLOSSARY

A.8.1 Values and other variables:

A		IO Use matrix; the use of commodities by each industry
A_j	$j \varepsilon I_{IND}$	Columns of **A**
A_{ij}	$i \varepsilon I_{COM}$ $j \varepsilon I_{IND}$	Share of input i in producing output j
BF		Net US private sector claims on rest-of-world
BF^{disc}		Stock-flow discrepancy in the US external accounts
BG		Government debt to domestic households
BG^{disc}		Stock-flow discrepancy in the US government accounts
*BG **		Government debt to rest-of-world
CA		Current account surplus of the US
CR		Households transfer to rest-of-world
EX_{it}		Exogenous projected exports.
GFI		Government net foreign investment
GINT		Government interest payments on public debt to households (including social insurance funds.)
$GINT^{rec}$		Arbitrage adjustment for interest income on government bonds
GINTR		Government interest payments to rest-of-world
GM		Government net imports
GR		Government transfers to rest-of-world
GTRAN		Government transfers to households
M_k		Expenditures by household k
MF		Full expenditures (including leisure)
SD^i	$i \varepsilon I_{COM}$	Shares of domestic output, imports in total supply of i
SF		Vector of shares of commodities and leisure in full consumption
SI^m	$m \varepsilon I_{INV}$	Shares of investment at node m
SM		Vector of shares of imports in total supply.
SN^m	$m \varepsilon I_{CNODE}$	Shares of consumption at node m
SP^{jm}	$j \varepsilon I_{IND}$ $m \varepsilon I_{PNODE}$	Shares of production at node m of industry j
S		Savings
TLUMP		Lump sum tax
TAXN		Non-tax receipts of the government
VC		Vector of values of household purchases of commodities
VFD		Vector of values of final demand for commodities
VG		Vector of values of government demand for commodities
VGG		Value of government purchases
VII		Value of domestic private investment
VI		Vector of values of investment inputs
VN		Vector of values of household purchases of NIPA commodities
VP^j	$j \varepsilon I_{IND}$	Vector of values of inputs into industry j
VQC		Vector of values of domestic commodity output

VQI		Vector of values (to producer) of domestic industry output
VQI^{tt}		Vector of values of domestic industry output inclusive of sales tax
VQS		Vector of values of total commodity supply
VX		Vector of values of commodity exports
W		Tangible wealth of private sector (households)
WF		Full wealth of private sector (households)
XR		Travel exports: Expenditures by foreign tourists in U.S.
Y		Income
YF		Full income (including imputations on leisure)
$YROW$		Net income from rest-of-world
$YROW^{rec}$		Arbitrage adjustment for income from rest-of-world
Y^*		Exogenous projected rest-of-world income
ΔG		Government deficit
ΔP^{BF}		Capital gains on net foreign assets
ΔP^{BG}		Capital gains on government bonds
ΔP^{BG*}		Capital gains on government liabilities to ROW
ψ^{I}		Aggregation constant of investment goods
ψ^{PK}		Aggregation constant of price of capital

A.8.2 Quantities:

CC		Aggregate consumption (commodities)
C^P		Vector of quantities of consumption of produced commodities.
C		Vector of consumption, commodities & non-produced goods.
C_i	$i \varepsilon I_{INP}$	Consumption of IO commodity i
F		Full consumption (commodities and leisure)
G^P		Vector of government purchases of commodities.
G		Vector of government purchases, commodities and non-produced goods.
G_i	$i \varepsilon I_{NCOM}$	Government purchases of commodity i
I^a		Aggregate investment in domestic capital stock
I		Vector of commodities used in aggregate investment.
I^m	$m \varepsilon I_{INV}$	Investment aggregate m
I_i^f	$i \varepsilon I_{NCOM}$	Investment of commodity i in fixed investment
I_i	$i \varepsilon I_{NCOM}$	Investment of commodity i in domestic capital stock
K		Capital stock located in the U.S.
KD		Quantity of total capital input normalized such that its rental price is one
KD_j	$j \varepsilon I_{NBUY}$	Quantity of capital input into sector j
LD_j	$j \varepsilon I_{NBUY}$	Quantity of labor input into sector j
$LEIS$		Leisure time
LH		Time endowment of economy
LS		Labor supply

M		Vector of competitive imports.
M_i	$i \varepsilon I_{COM}$	Imports of (competitive) commodities
N^{eq}		Number of household equivalent members in economy
N^m	$m \varepsilon I_{CNODE}$	Consumption of NIPA aggregate m
N_i	$i \varepsilon I_{PCE}$	Consumption of NIPA commodities
NCI_j	$j \varepsilon I_{NBUY}$	Non-competitive imports into sector j
QC_i	$i \varepsilon I_{COM}$	Total domestic output of commodity i
QI_j	$j \varepsilon I_{IND}$	Output of industry j
QP^{jm}	$j \varepsilon I_{IND} \quad m \varepsilon I_{PNODE}$	Aggregate input m into industry j
QP_i^j	$i \varepsilon I_{COM} \quad j \varepsilon I_{IND}$	Input of commodity i into industry j
QS_i	$i \varepsilon I_{COM}$	Total supply of commodity i
X		Vector of exports.
X_i	$i \varepsilon I_{COM}$	Exports of commodity i
X_i^{tr}	$i \varepsilon I_{COM}$	Travel exports of commodity i

A.8.3 Prices:

e		"exchange rate"
i^*		interest rate on private US owned foreign assets
r		After tax interest rate
w		Price of total hours (work and leisure)
$\bar{w}$		After tax average wage rate
w^{LE}		Price of leisure
P_i^*	$i \varepsilon I_{COM}$	World price for US exports.
P^{Hm}	$m \varepsilon I_{CNODE}$	Vector of prices at node m of consumption function
P^{im}	$m \varepsilon I_{INV}$	Vector of prices at node m of investment function
P^{Pjm}	$j \varepsilon I_{IND} \quad m \varepsilon I_{PNODE}$	Vector of prices at node m of industry j's production function
PC_i	$i \varepsilon I_{COM}$	Price of domestically produced commodities
PF		Price of full consumption
PI_j	$j \varepsilon I_{IND}$	Price of industry output paid by buyers
PII		Price of aggregate investment goods
PII^m	$m \varepsilon I_{INV}$	Price of investment aggregate m.
PII_{mi}	$mi \varepsilon I_{INVm}$	Union of above aggregate investment prices and supply prices.
PK		Price of capital stock
PKD_j	$j \varepsilon I_{BUY}$	Rental price of capital paid by producer
PLD_j	$j \varepsilon I_{BUY}$	Price of labor paid by employers
PM_i	$i \varepsilon I_{COM}$	Price of non-competitive imports paid by importers
PN_n	$n \varepsilon I_{NIPA}$	Price of NIPA PCE commodity
PN^m	$m \varepsilon I_{CNODE}$	Price of consumption aggregate m
PN_{mi}	$mi \varepsilon I_{CNODEm}$	Union of above 2 sets of consumption prices
$PNCI_i$	$j \varepsilon I_{BUY}$	Price of imports paid by importers
PO_j		Price of industry output received by producer
PP^{jm}	$j \varepsilon I_{IND} \quad m \varepsilon I_{PNODE}$	Price of aggregate input m into industry j

PP^{j}_{mi}	$mi \varepsilon I_{PNODEm}$	Union of above set of aggregate production prices and prices of inputs.
PS		Vector of supply prices.
PS_i	$i \varepsilon I_{COM}$	Price of commodities to buyers
PS^H		Vector of supply prices and prices of non-produced items consumed by household sector.

A.8.4 Parameters of behaviorial equations:

ρ		Pure rate of time preference
σ		Household intertemporal elasticity of substitution
α^F		Shares (at unit prices) of commodities and leisure in full consumption
B^F		Share elasticity of components of full consumption (w.r.t. prices)
α^F_0		Shares (at $t = -\infty$) of commodities and leisure in F
β^F_0		Trend coefficient of commodities and leisure in F
μ^F		Slope of logistic curve in F
τ^F		Mid-point of logistic curve
α^{Hm}	$m \varepsilon I_{CNODE}$	Shares (at unit prices) of consumption at node m
B^{Hm}		Share elasticity of consumption (w.r.t. prices) at node m
B_{pA}		Coefficients on demographic characteristics of CC function
ξ^d		Distribution coefficient of CC function
ξ^L		Vector of demographic dummies
α^j_0	$j \varepsilon I_{IND}$	Cost function constant
α^{Pjm}	$j \varepsilon I_{IND}$ $m \varepsilon I_{PNODE}$	Shares (at unit prices) of of inputs into industry j at node m
B^{Pjm}		Share elasticity of input demands (w.r.t. prices) at node m
B^j_{pt}	$j \varepsilon I_{IND}$	Biases of technical change
μ^j		Slope of logistic curve representing index of technology
τ^j		Mid-point of logistic curve
α^{IY}		Share of inventory investment in total investment
α^{IY}_i	$i \varepsilon I_{COM}$	Share of inventory investment going to commodity i
α^{Im}	$m \varepsilon I_{INV}$	Shares (at unit prices) of commodities at investment node m
B^{Im}	$m \varepsilon I_{INV}$	Shares elasticity of components of totalinvestment at node m
α^I		Constant term in logistic curve of aggregation constant converting investment into capital, ψ^I
β^I		Trend coefficient of logistic curve, ψ^I
μ^I		Slope of logistic curve, ψ^I
τ^I		Mid-point of logistic curve, ψ^I

α^{PK}		Constant term in logistic curve of aggregation constant converting price of investment into price of capital, ψ^{PK}
β^{PK}		Trend coefficient of logistic curve, ψ^{PK}
μ^{PK}		Slope of logistic curve, ψ^{PK}
τ^{PK}		Mid-point of logistic curve, ψ^{PK}
α^{Mi}	$i \varepsilon I_{COM}$	Shares (at unit prices) of domestic commodities and imports in total supply
α_0^{Mi}		Shares (at t=$-\infty$) of domestic commodities and imports
β_0^{Mi}		Trend coefficient of domestic commodities and imports
μ^{Mi}		Slope of logistic curve representing import penetration
τ^{Mi}		Mid-point of logistic curve
B^{Mi}	$i \varepsilon I_{COM}$	Shares elasticity of components of total supply
η^{i}	$i \varepsilon I_{COM}$	Export price elasticities
α_i^G	$i \varepsilon I_{INP}$	Share of government expenditures on i
δ		Rate of depreciation of capital stock
ψ^K		Aggregation constant of capital services
ψ_j^K	$j \varepsilon I_{BUY}$	Aggregation constants of capital
ψ_j^L	$j \varepsilon I_{BUY}$	Aggregation constants of labor
ψ_C^L		Aggregation constants of leisure
H		Bridge matrix linking NIPA "Personal Cons. Expenditures" commodities to IO commodities
M		IO Make matrix; the contribution of each industry to

A.8.5 Tax rates:

tk		Tax rate on capital income
tl^a		Average tax rate on labor income
tl^m		Marginal tax rate on labor income
tp		Property tax rate
tr_i	$i \varepsilon I_{COM}$	Tariff rate on competitive imports
tr_i^n	$i \varepsilon I_{BUY}$	Tariff rate on non-comp imports
tr_i *	$i \varepsilon I_{COM}$	World tariff rate on US exports
tt_j	$j \varepsilon I_{IND}$	Indirect business tax (sales tax)
tw		Wealth tax rate (estate taxes)