Appendix A. Equations of the Model & Glossary (Version 10)

A.0 Notation:

Time
t ε I_T $I_T = \{1, 2, \ldots T, \ldots\}$

Industry/Producer
j ε I_{IND} $I_{IND} = \{1, 2, \ldots, 35\}$

IO Commodities
i ε I_{COM} $I_{COM} = \{1, 2, \ldots, 35\}$

Industry Inputs
i ε I_{INP} $I_{INP} = \{1, 2, \ldots, 35, NCI, K, L\}$

NIPA PCE Commodities
n ε I_{PCE} $I_{PCE} = \{1, 2, \ldots, 38\}$

Purchasers of domestic output
j ε I_{BUY} $I_{BUY} = \{1, 2, \ldots, 35, C, I, G, X\}$

Households
k ε I_{POP}

Nodes of production function
m ε I_{PNODE} $I_{PNODE} = \{EN, M, \ldots . WP\}$
i ε I_{PNODEm} I_{PNODEm} in Table 2.4

Nodes of consumption function
m ε I_{CNODE} $I_{CNODE} = \{EN, FD, \ldots . RS\}$
i ε I_{CNODEm} I_{CNODEm} in Table 2.3

Nodes of investment function
m ε I_{INV} $I_{INV} = \{fixed, \ldots, mining\}$
i ε I_{INVm} I_{INVm}

Externalities
x ε I_{EXT} $I_{EXT} = \{1, 2, 3, 4\}$

Vector of 1's
ι

Transpose of matrix A
A'

Diagonal matrix of a vector v
$Diag(v)$

A.1 Household Sector

Household first stage decision; Euler equation.

$$\text{Max} \sum_{t=1}^{\infty} \frac{N_t^{eq}}{(1+\rho)^t} (F_t/N_t^{eq})^{1-\frac{1}{\sigma}} \tag{A.1.1}$$

subject to

$$WF \equiv PK_0K_0 + BG_0 + BF_0 + \sum_{t=1}^{\infty} \frac{\bar{w}_t LH_t + misc_t}{\prod_{s=1}^{t} 1+r_s} \geq \sum_{t=1}^{\infty} \frac{PF_t F_t}{\prod_{s=1}^{t} 1+r_s} \tag{A.1.2}$$

where

$$misc = GTRAN + (1-tk)(GINT^{rec} + YROW^{rec}) - twW_{t-1} - TLUMP - CR - TAXN$$

$$\left[\frac{F_t/N_t^{eq}}{F_{t-1}/N_{t-1}^{eq}} \right]^{\frac{1}{\sigma}} = \frac{1+r_t}{1+\rho} \frac{PF_{t-1}}{PF_t} \tag{A.1.3}$$

Household second stage decision, goods-leisure choice.

$F_t = F(CC_t, LEIS_t)$ Formulated with the price dual: (A.1.4)

$$VV(PCC, w^{LE}, MF) = \max \; F(CC, LEIS) \qquad s.t. \;\; MF = PCC.CC + w^{LE}.LEIS$$

$$-\ln VV = \alpha^{F\prime} \ln \frac{PF^F}{MF} + \tfrac{1}{2} \ln \frac{PF^{F\prime}}{MF} B^F \ln \frac{PF^F}{MF} \tag{A.1.5}$$

$$= \alpha^{F\prime} \ln PF^F + \tfrac{1}{2} \ln PF^{F\prime} B^F \ln PF^F - \ln MF$$

$$\ln PF^F \equiv (PCC, w^{LE})'$$

$$\ln PF = \alpha^F \ln PF^F + \tfrac{1}{2} \ln PF^{F\prime} B^F \ln PF^F \tag{A.1.6}$$

$$= (\alpha_0^F + \frac{\beta_0^F}{1+\exp(-\mu^F(t-\tau^F)}) \ln PF^F + \tfrac{1}{2} \ln PF^{F\prime} B^F \ln PF^F$$

$$SF \equiv \begin{bmatrix} PCC.CC/MF \\ w^{LE}.LEIS/MF \end{bmatrix} = \alpha^F + B^F \ln PF^F \tag{A.1.7}$$

$$CC = SF_1 * MF/PCC \tag{A.1.8}$$

$$MF = PF.F \tag{A.1.9}$$

$$= PCC.CC + w^{LE}.LEIS \tag{A.10}$$

$$LS = LH - \psi_C^L LEIS \tag{A.1.11}$$

$$\bar{w} LH = (1 - tl^a) \frac{wLS}{1 - tl^m} + w\psi_C^{LE} LEIS$$

Income and savings:

$$W_t \equiv PK_t K_t + BG_t + BF_t \tag{A.1.12}$$

$$YF_t = capital\_income + \bar{w}_t LH_t + GTRAN_t - TLUMP_t - twW_{t-1} \tag{A.1.13}$$

$$Y_t = capital\_income + w_t LS_t \frac{1 - tl^a}{1 - tl^m} \tag{A.1.14}$$

$$+ GTRAN_t - TLUMP_t - twW_{t-1}$$

$$capital\_income = \sum_j cash\_flow_j + PKD_h KD_h + t_h BH - t_h^p PIIK_{ht-1}$$

$$+ (1 - t_n^{e)} (i_t BF_{t-1} - YROW_t^{rec} + i_t BG_{t-1} - GINT_t^{rec})$$

where $cash\_flow_j$ is (A.3.48); BH is (A.3.51).

$$S_t = YF_t - PF_t F_t - CR_t - TAXN_t + \theta PII_t I_t^a \tag{A.1.15}$$

$$= YF_t - PCC_t CC_t - w_t LH_t - CR_t - TAXN_t$$

$$= Y_t - PCC_t CC_t - CR_t - TAXN_t$$

Household third stage decision, allocation of consumption goods:

NESTED STRUCTURE OF CONSUMPTION (A.1.16)

$CC = CC(N^{EN}, N^{FD}, N^{ND}, N_K, N^{SV})$	Commodity aggregate
$N^{EN} = N^{EN}(N_6, N^{FL}, N_{18}, N_{19})$	Energy
$N^{FD} = N^{FD}(N_1, N_2, N_3, N_9)$	Food agg.
$N^{ND} = N^{ND}(N^{CS}, N^{HA}, N_{12}, N^{NM})$	Non-durables
$N^{SV} = N^{SV}(N^{HR}, N^{HS}, N^{TR}, N^{MD}, N^{SM})$	Services agg.
$N^{FL} = N^{FL}(N_7, N_8)$	Fuel and wood
$N^{CS} = N^{CS}(N_4, N_5)$	Clothing and shoes
$N^{HA} = N^{HA}(N_{10}, N_{11})$	Household articles
$N^{NM} = N^{NM}(N_{13}, N_{14}, N_{15}, N_{16})$	Miscellaneous non-durables
$N^{HR} = N^{HR}(N_{17}, N_{34})$	Rental housing

$N^{HS} = N^{HS}(N_{20}, N_{21}, N_{22}, N_{23})$ Household services
$N^{TR} = N^{TR}(N_{24}, N_{25})$ Transportation
$N^{MD} = N^{MD}(N_{26}, N_{27})$ Medical
$N^{SM} = N^{SM}(N_{28}, N^{BS}, N^{RS}, N_{32})$ Miscellaneous services

$N^{BS} = N^{BS}(N_{29}, N_{30})$ Business services
$N^{RS} = N^{RS}(N_{31}, N_{33})$ Recreation

subscripts $\varepsilon\ I_{PCE}$

Dual of **top** (m=1) tier consumption demands CC=CC(...) :

$$\ln V_k = \alpha^{H1} \ln P^{H1} + \tfrac{1}{2} \ln P^{H1}\ B^H \ln P^{H1} + (1 - \ln P^{H1} B\iota) \ln M_k + \ln P^{H1} B_{pA}\, A_k \qquad \text{(A.1.17)}$$

$$\ln P^{H1} = (\ln PN^{EN}, \ln PN^{FD}, PN^{ND}, PN^{K}, PN^{SV}) \qquad k\ \varepsilon\ I_{POP}$$

$$\ln PCC = \iota' B^H \ln P^{H1} \ln \frac{PCC.CC}{N^{eq}} - \alpha^H \ln P^{H1} + \tfrac{1}{2} \ln P^{H1} B^H \ln P^{H1} \qquad \text{(A.1.18)}$$

$$SN^{TOP} \equiv \left(\frac{PN^{EN} N^{EN}}{PCC.CC}, \frac{PN^{FD} N^{FD}}{PCC.CC}, \frac{PN^{ND} N^{ND}}{PCC.CC}, \frac{PN^{K} N^{K}}{PCC.CC}, \frac{PN^{SV} N^{SV}}{PCC.CC} \right)' \qquad \text{(A.1.19)}$$

$$= \frac{\alpha^H + B^H \ln P^{H1} - B^H \iota\, \xi^d + B_{pA}\, \xi^L}{-1 + \iota' B^H \ln P^{H1}}$$

Price dual of **lower** tiers consumption demands $N^m(\ldots)$:

$$\ln PN^m = \alpha^{Hm\prime} \ln P^{Hm} + \tfrac{1}{2} \ln P^{Hm\prime}\, B^{Hm} \ln P^{Hm} \qquad m\ \varepsilon\ I_{CNODE} \qquad \text{(A.1.20)}$$

$$\ln P^{Hm} \equiv (\ln PN_{m1}, .., \ln PN_{mi}, .., \ln PN_{m,im})' \qquad i\ \varepsilon\ I_{CNODEm} \qquad \text{(A.1.21)}$$

$$SN^m = \begin{bmatrix} PN_{m1} N_{m1} / PN^m N^m \\ \cdots \\ PN_{m,im} N_{m,im} / PN^m N^m \end{bmatrix} = \alpha^{Hm} + B^{Hm} \ln PN^{Hm} \quad \begin{matrix} m\ \varepsilon\ I_{CNODE} \\ mi\ \varepsilon\ I_{CNODEm} \end{matrix} \qquad \text{(A.1.22)}$$

$$PN_{mi}\ \varepsilon\ \{PN_1, \ldots, PN_{34,}\ PN^{EN}, \ldots, PN^{RS}\}$$

$$N_{mi}\ \varepsilon\ \{N_1, \ldots, N_{34,}\ N^{EN}, \ldots, N^{RS}\}$$

$$PN_1 N_1 = SN_1^{FD} * SN_2^{TOP} * PCC.CC \qquad \text{(A.1.23)}$$

$$PN_2N_2 = SN_2^{FD} * SN_2^{TOP} * PCC.CC$$

. . . . . .

$$PN_{34}N_{34} = SN_2^{HR} * SN_1^{SV} * SN_5^{TOP} * PCC.CC$$

$$VN \equiv (\, PN_1N_1, \ldots, PN_{34}N_{34}, PKD_CKD_C \,)'$$

$$PN = \mathbf{H}'\, PS^C \qquad \textit{where the components of } PS^C: \tag{A.1.24}$$

$$PS_i^C = (1 + tc_i)PS_i \qquad i\ \varepsilon\ I_{COM} \tag{A.1.25}$$

$$PS_N^C = (1 + tc_N)PNCI_C$$

$$PS_K^C = (1 + tc_K)PKD_C$$

$$PS_L^C = (1 + tc_L)PLD_C$$

Converting from NIPA categories to IO categories:

$$VC \equiv (\, PS_1C_1, \ldots, PS_{35}C_{35}, \ldots, PLD_CLD_C \,)' \tag{A.1.26}$$

$$= \mathbf{H}\, VN$$

$$C_i = VC_i/PS_i \qquad i\ \varepsilon\ I_{INP} \tag{A.1.27}$$

$$C^P \equiv (C_1, C_2, \ldots, C_{35})'$$

$$C \equiv (C_1, \ldots, C_{35}, NCI_C, KD_C, LD_C)'$$

A.2 Producer model

NESTED STRUCTURE OF PRODUCTION (A.2.1)

$QI_j = QI^j(KD_j, LD_j, QP^{jE}, QP^{jM})$ Industry output

$QP^{jE} = QP^E(QP^j_3, QP^j_4, QP^j_{16}, QP^j_{30}, QP^j_{31})$ Energy aggregate
$QP^{jM} = QP^M(QP^j_6, QP^{jMA}, QP^{jME}, QP^{jMN}, QP^{jMS})$ Material aggregate

$QP^{jMA} = QP^{AG}(QP^j_1, QP^j_7, QP^j_8, QP^{jTX}, QP^{jWP})$ Agriculture products
$QP^{jME} = QP^{ME}(QP^{jFM}, QP^{jMC}, QP^{jEQ})$ Metallic agg.
$QP^{jMN} = QP^{MN}(QP^j_5, QP^j_{15}, QP^j_{17}, QP^j_{19}, QP^j_{27})$ Non-metallic agg.
$QP^{jMS} = QP^{MS}(QP^j_{28}, QP^j_{32}, QP^j_{33}, QP^j_{34}, QP^{jOS})$ Service agg.

$QP^{jTX} = QP^{TX}(QP^j_9, QP^j_{10}, QP^j_{18})$ Textile agg.
$QP^{jWP} = QP^{WP}(QP^j_{11}, QP^j_{12}, QP^j_{13}, QP^j_{14})$ Wood and paper agg.
$QP^{jFM} = QP^{FM}(QP^j_2, QP^j_{20}, QP^j_{21})$ Primary metal agg.
$QP^{jMC} = QP^{MC}(QP^j_{22}, QP^j_{23})$ Machinery agg.
$QP^{jEQ} = QP^{EQ}(QP^j_{24}, QP^j_{25}, QP^j_{26})$ Equipment agg.
$QP^{jOS} = QP^{OS}(QP^j_{29}, QP^j_{35}, QP^j_{NCI})$ Miscellaneous services

Price dual of **top** tier of production functions $QI_j = QI(\dots)$:

$PO_j = PO^j(PKD_j, PLD_j, PP^{jE}, PP^{jM}) \qquad j \varepsilon I_{IND}$

$$\ln PO_j = \alpha_0^j + \alpha^{Pj\prime} \ln P^{Pj0} + \tfrac{1}{2} \ln P^{Pj0\prime} B^{Pj} \ln P^{Pj0} + \ln P^{Pj0\prime} B^j_{pt} g(t) + \alpha_t^j\, g(t) + \tfrac{1}{2}\, \beta^j_{tt} g(t)^2 + \ln \lambda_j \qquad \text{(A.2.2)}$$

where $g(t) = \dfrac{1}{1+\exp(-\mu^j(t-\tau^j))}$.

and $\ln P^{Pj0} \equiv (\ln PKD_j, \ln PLD_j, \ln PP^{jEN}, \ln PP^{jM})'$

$$SP^{jTOP} = \begin{bmatrix} PKD_j KD_j / PO_j QI_j \\ .. \\ PP^{jM} QP^{jM} / PO_j QI_j \end{bmatrix} = \alpha^{Pj} + B^{Pj} \ln P^{Pj0} + B^j_{pt}\, g(t) \qquad \text{(A.2.3)}$$

Price dual of **lower** tiers of production functions, $QP^{jm} = QP(\dots)$:

$$\ln PP^{jm} = \alpha_0^{jm} + \alpha^{Pjm\prime} \ln P^{Pjm} + \tfrac{1}{2} \ln P^{Pjm\prime} B^{Pjm} \ln P^{Pjm} \qquad m \varepsilon I_{PNODE} \qquad \text{(A.2.4)}$$

$$\ln P^{Pjm} \equiv (\ln PP^{j}_{m1}, .., \ln PP^{j}_{mi}, .., \ln PP^{j}_{m,im})' \qquad i \varepsilon I_{PNODEm}$$

$$SP^{jm} = \begin{bmatrix} PP^{j}_{m1} QP^{j}_{m1} / PP^{jm} QP^{jm} \\ .. \\ PP^{j}_{m,im} QP^{j}_{m,im} / PP^{jm} QP^{jm} \end{bmatrix} = \alpha^{Pjm} + B^{Pjm} \ln P^{Pjm} \qquad \text{(A.2.5)}$$

$$PP^{j}_{mi} \; \varepsilon \; \{PS_1, \ldots, PS_{35}, PNCI_j, PP^{jMA}, \ldots, PP^{jOS}\}$$

$$QP^{j}_{mi} \; \varepsilon \; \{QP^{j}_1, \ldots, QP^{j}_{35}, NCI_j, QP^{jMA}, \ldots, QP^{jOS}\}$$

$$VP^{j} \equiv (\, PS_1 QP^{j}_1, \ldots PS_{35} QP^{j}_{35}, PNCI_j NCI_j, \, PKD_j KD_j, PLD_j LD_j \,)'$$

$$VQI \equiv (\, PO_1 QI_1, \ldots, PO_{35} QI_{35} \,)'$$

Taxes. (Net vs. Gross Output)

$$PI_j = (1 + tt_j + tx^{v}_{j}) PO_j + tu_j + tx^{u}_{j} \qquad j \; \varepsilon \; I_{IND} \qquad \text{(A.2.6)}$$

$$VQI^{tt} \equiv (\, PI_1 QI_1, \ldots, PI_{35} QI_{35} \,)'$$

$$= Diag(\iota + tt^{full})\, VQI$$

Commodities from industry outputs:

$$PC = \mathbf{M}'\, PI \qquad \text{(A.2.7)}$$

$$VQC \equiv (\, PC_1 QC_1, \ldots, PC_{35} QC_{35} \,)' \qquad \text{(A.2.8)}$$

$$= \mathbf{M}'\, VQI^{tt}$$

$$QC_i = VQC_i / PC_i \qquad i \; \varepsilon \; I_{COM} \qquad \text{(A.2.9)}$$

The input-output USE matrix:

$$A_{1j} = SP^{jAG}_1 * SP^{jM}_2 * SP^{jTOP}_4 \qquad \text{(A.2.10)}$$

$$A_{2j} = SP^{jFM}_1 * SP^{jME}_1 * SP^{jM}_3 * SP^{jTOP}_4$$

. . . . . .

$$A_{35j} = SP^{jOS}_2 * SP^{jMS}_5 * SP^{jM}_5 * SP^{jTOP}_4$$

$$A_j \equiv (A_{1j}, A_{2j}, \ldots, A_{35j})' \qquad j \varepsilon I_{IND} \tag{A.2.11}$$

$$\mathbf{A} = [A_1, A_2, \ldots, A_{35}] \tag{A.2.12}$$

$$PNCI_j NCI_j = SP_3^{jOS} * SP_5^{jMS} * SP_5^{jM} * SP_4^{jTOP} * PO_j * QI_j \tag{A.2.13}$$

$$PKD_j KD_j = SP_1^{jTOP} * PO_j * QI_j \tag{A.2.14}$$

$$PLD_j LD_j = SP_2^{jTOP} * PO_j * QI_j \tag{A.2.15}$$

$$KD_4 = \overline{KD}_4 \qquad \text{(oil sector)} \tag{A.2.16}$$

A.3 Capital and Investment

The Bank as owner of economy aggregate capital.

$$\text{Max} \sum_{t=u}^{\infty} \frac{(1-tk)(PKD_t \psi^K K_{t-1} - tpPK_{t-1}K_{t-1}) - (1-\theta)PII_t I_t^a}{\prod_{s=u}^{t} 1+r_s} \tag{A.3.1}$$

subject to

$$K_t = (1-\delta)K_{t-1} + \psi^I \, I_t^a \tag{A.3.2}$$

Hamiltonian :

$$\frac{(1-tk)(PKD_t \psi^K K_{t-1} - tpPK_{t-1}K_{t-1}) - (1-\theta)PII_t I_t^a}{\prod_{s=u}^{t} 1+r_s} + \frac{\lambda_t}{\prod_{s=u}^{t} 1+r_s} \left((1-\delta)K_{t-1} + \psi^I I_t^a - K_t \right) \tag{A.3.3}$$

Euler equation :

$$(1+r_t)\frac{PII_{t-1}}{\psi_{t-1}^I} = \frac{1-tk}{1-\theta}(PKD_t \psi_t^K - tp\, PK_{t-1}) + (1-\delta)\frac{PII_t}{\psi_t^I} \tag{A.3.4}$$

Aggregation relationships.

$$PK_t = \psi_t^{PK} \; PII_t(1-\theta) \tag{A.3.5}$$

$$KD_t = \psi_t^K \; K_{t-1} \tag{A.3.6}$$

$$\psi_t^I = \alpha^I + \frac{\beta^I}{1+\exp(-\mu^I(t-\tau^I))} \tag{A.3.7}$$

$$\psi_t^{PK} = \alpha^{PK} + \frac{\beta^{PK}}{1+\exp(-\mu^{PK}(t-\tau^{PK}))} \tag{A.3.8}$$

$$VII = PII \,.\, I^a \tag{A.3.9}$$

NESTED STRUCTURE OF INVESTMENT (A.3.10)

$I^a = I^a(I^{fixed}, I^{inventory})$	Aggregate investment
$I^{fixed} = I^{FX}(I^{long}, I^{short})$	Fixed investment agg.
$I^{inventory} = I^{IY}$	Change in business inventories
$I^{long} = I^{LG}(I_6, I_{33})$	Long-lived investment agg.
$I^{short} = I^{SH}(I^{vehicles}, I^{machinery}, I^{services})$	Short-lived investment agg.
$I^{vehicles} = I^{VE}(I_{24}, I_{25})$	Vehicle agg.
$I^{machinery} = I^{MC}(I_{22}, I_{23}, I^{other-m})$	Machinery agg.
$I^{services} = I^{SV}(I_{32}, I^{other-s})$	Services agg.
$I^{other-m} = I^{MO}(I^{gadgets}, I^{wood}, I^{nonmetal}, I^{misc})$	Other machinery agg.
$I^{other-s} = I^{SO}(I_{34}, I^{movers})$	Other services agg.
$I^{gadgets} = I^{GD}(I_{20}, I_{21}, I_{26})$	Metals and instruments agg.
$I^{wood} = I^{WD}(I_{11}, I_{12})$	Wood products agg.
$I^{nonmetal} = I^{MN}(I_{15}, I_{17}, I_{19}, I_{27})$	Nonmetallic products agg.
$I^{misc} = I^{OO}(I^{textile}, I_{13}, I^{mining})$	Miscellaneous agg.
$I^{mover} = I^{TC}(I_{28}, I_{29})$	Transportation and Communications agg.
$I^{textile} = I^{TX}(I_9, I_{10}, I_{18}, I_{NCI})$	Textile agg.
$I^{mining} = I^{MG}(I_2, I_4)$	Minerals agg.

At top tier of investment functions I=I(...) :

$$VII = VII^{fixed} + VII^{invy} \tag{A.3.11}$$

$$\frac{VII^{invy}}{VII} = \alpha^{IY} \tag{A.3.12}$$

Price dual of investment demand tiers $I^m = I^m(\dots)$:

$$VI_i^{invy} = \alpha_i^{IY}\, VII^{invy} \qquad i \varepsilon I_{COM} \tag{A.3.13}$$

$$\ln PII^m = \alpha^{\mathrm{Im}} \ln P^{\mathrm{Im}} + \tfrac{1}{2} \ln P^{\mathrm{Im}\prime} B^{\mathrm{Im}} \ln P^{\mathrm{Im}} + \log \lambda^I \qquad m \varepsilon I_{INV} \tag{A.3.14}$$

$$\ln P^{\mathrm{Im}} \equiv (\ln PII_{m1}, \dots, \ln PII_{mi}, \dots, \ln PII_{m,im})' \qquad i \varepsilon I_{INVm}$$

$$SI^m = \begin{bmatrix} PII_{m1} I^f_{m1} / PII^m I^m \\ \cdots \\ PII_{m,im} I^f_{m,im} / PII^m I^m \end{bmatrix} = \alpha^{\mathrm{Im}} + B^{\mathrm{Im}} \ln PII^{\mathrm{Im}} \qquad \begin{matrix} m \ \varepsilon \ I_{INV} \\ mi \ \varepsilon \ I_{INVm} \end{matrix} \qquad \text{(A.3.15)}$$

$$PII_{mi} \ \varepsilon \ \{PS_1, \ldots, PS_{35}, PII^{fixed}, \ldots, PII^{mining}\}$$

$$I_{mi} \ \varepsilon \ \{I^f_1, \ldots, I^f_{35}, I^{fixed}, \ldots, I^{mining}\}$$

Values of individual commodities making up aggregate investment demand:

$$VI_1 = 0 + VI^{invy}_1 \qquad \text{(A.3.16)}$$

$$VI_2 = SI^{MG}_1 * SI^{OO}_3 * SI^{MO}_4 * SI^{MC}_3 * SI^{SH}_2 * SI^{FX}_2 \ VII^{fixed} + VI^{invy}_2$$

. . . . . .

$$VI_{34} = SI^{SO}_1 * SI^{SV}_2 * SI^{SH}_3 * SI^{FX}_2 \ VII^{fixed} + VI^{invy}_{34}$$

$$VI_{35} = 0$$

$$I_i = VI_i / PS_i \qquad \text{(A.3.17)}$$

$$VI \equiv (PS_1 I_1, \ldots, PS_{35} I_{35}, PNCI_I \, NCI_I)'$$

$$I^P \equiv (I_1, \ldots, I_{35})'$$

$$I \equiv (I_1, \ldots, I_{35}, NCI_I)'$$

The Bank as owner of disaggregated capital

Capital services

$$KD = KD_1 + \cdots + KD_{35} + KD_h \qquad \text{(A.3.18)}$$

$$KD_j = KD^j(KD_{jc}, KD_{jn}) \qquad corporate, noncorporate \qquad j \varepsilon I_{IND} \qquad \text{(A.3.19)}$$

$$KD_{jc} = KD(KD_{jcs}, KD_{jcl}) \qquad short, long-lived \ assets \qquad c = c, n \qquad \text{(A.3.20)}$$

$$VKD_{jcst} = PKD_{jcst} KD_{jcst} \qquad value \ of \ capital \ services \qquad \text{(A.3.21)}$$

* Industry capital

$$\ln PKD_j = \alpha^j_{KD0} + \alpha^j_{KD} \ln P + \tfrac{1}{2} \ln P' B^j_{KD} \ln P, \qquad \ln P = (\ln PKD_{jc}, \ln PKD_{jn})' \qquad \text{(A.3.22)}$$

$$\begin{bmatrix} PKD_{jc} KD_{jc} / VKD_j \\ PKD_{jn} KD_{jn} / VKD_j \end{bmatrix} = \alpha^j_{KD} + B \ln P, \qquad \text{(A.3.23)}$$

* Corporate capital

$$\ln PKD_{jc} = \alpha^{c}_{KD0} + \alpha^{jc}_{KD}{}' \ln P + \tfrac{1}{2} \ln P' B^{jc}_{KD} \ln P \qquad \ln P = (\ln PKD_{jcs}, \ln PKD_{jcl})' \tag{A.3.24}$$

$$\begin{bmatrix} PKD_{jcs} KD_{jcs}/VCD_j \\ PKD_{jcl} KD_{jcl}/VQD_j \end{bmatrix} = \alpha^{jc}_{KD} + B^{jc}_{KD} \ln P, \tag{A.3.25}$$

$$PKD_{jcst} = [\frac{1 - ITC_{cs} - t_c z_{cs}}{1 - t_c} (r_{jc} + (1+\pi)\delta_{cs}) + \gamma^p_c t^p_c] PK_{t-1} \qquad s = s, l \tag{A.3.26}$$

NOTE: We do not index PK by PK_{jcs} or PK_{jns}, but have a common price of capital goods for all buyers. Similarly, π is common to all equations.

$$r_{jc} = (1 - \beta_{jc}) r^{equ}_c + \beta_{jc}(1 - (1 - \gamma^i_c) t_c) i_t - \pi \tag{A.3.27}$$

where $r^{equ}_c = \dfrac{\rho - \pi[1 - (1 - \gamma^g_c) t^g_c]}{1 - t^{earn}_c} (1 - \alpha^{DIV} \gamma^d_c t_c)$

where $t^{earn}_c = \alpha^{DIV} t^e_c + (1 - \alpha^{DIV}) t^g_c$

* Noncorporate capital

$$\ln PKD_{jn} = \alpha^{jn}_{KD0} + \alpha^{jn}_{KD} \ln P + \tfrac{1}{2} \ln P' B^{jn}_{KD} \ln P, \qquad \ln P = (\ln PKD_{jns}, \ln PKD_{jnl})' \tag{A.3.28}$$

$$PKD_{jns} = [\frac{1 - ITC_{ns} - t^e_n z_{ns}}{1 - t^e_n} (r_{jn} + (1+\pi)\delta_{ns}) + \gamma_n t^p_n] PK_{t-1} \qquad s = s, l \tag{A.3.29}$$

$$\begin{bmatrix} PKD_{ns} KD_{ns}/VND_j \\ PKD_{nl} KD_{nl}/VND_j \end{bmatrix} = \alpha^{jn}_{KD} + B^{jn}_{KD} \ln P, \tag{A.3.30}$$

$$r_{jn} = (1 - \beta_{jn})(\rho - \pi[1 - (1 - \gamma^g_n) t^g_n]) + \beta_{jn}(1 - (1 - \gamma^i_n) t^e_n) i_t - \pi \tag{A.3.31}$$

* Household capital

$$PKD_{hs} = [r_h + (1+\pi)\delta_{hs} + (1 - \gamma^p_h) t^p_{hs}] PK_{t-1} \qquad s = s, l \tag{A.3.32}$$

$$\ln PKD_h = \alpha^h_{KD0} + \alpha^h_{KD} \ln P + \tfrac{1}{2} \ln P' B^h_{KD} \ln P \qquad \ln P = (\ln PKD_{hs}, \ln PKD_{hl})' \tag{A.3.33}$$

$$\begin{bmatrix} PKD_{hs} KD_{hs}/VHD \\ PKD_{hl} KD_{hl}/VHD \end{bmatrix} = \alpha^h_{KD} + B^h_{KD} \ln P, \tag{A.3.34}$$

$$r_h = (1 - \beta_h) r^{equ}_h + \beta_h (1 - (1 - \gamma^i_h) t^e_h (1 - dhi)) i_t - \pi \tag{A.3.35}$$

where $r^{equ}_h = \rho - \pi[1 - (1 - \gamma^g_n) t^g_h]$

* Short, long

In data construction, short and long capital is aggregated over the corresponding components from the list of 51 BEA assets classes; $K_{jcst} = \sum_i K_{ijcst}$ and $K_{jclt} = \sum_i K_{ijclt}$ for $j = 1, .. 35$ and $c = c, l, h$.

In this version of the model we do not keep track of $K_{ijcst} = (1 - \delta_i) K_{ijcst-1} + I_{ijcst}$. In other

words there is no effort to link the investment column of the IO table (A.3.17) and I_{ijcst}.

$$i_t = constant * PKD_t - equity\ \ premium \tag{A.3.36}$$

$$t_c = t_c^f(1 - t_c^s) + t_c^s \tag{A.3.37}$$

Value of Depreciation deductions and property tax

$$DC_j = \sum_{s,l} z_{cs}(r_c + (1+\pi)\delta_{cs})(1 - ITC_{cs} - t_c z_{cs})PK_{t-1}K_{jcst-1} \tag{A.3.38}$$

$$DN_j = \sum_{s,l} z_{ns}(r_n + (1+\pi)\delta_{ns})(1 - ITC_{ns} - t_n z_{ns})PK_{t-1}K_{jnst-1} \tag{A.3.39}$$

$$R_{jc}^p = t_c^p(1 - ITC_{cs} - t_c z_{cs})PK_{t-1}K_{jcst-1} \qquad c = c, n \quad j = 1\varepsilon I_{IND} \tag{A.3.40}$$

Corporate income tax base; interest part, equity part

$$IC_{cj} = \beta_{jc}\sum_{s,l}(1 - ITC_{cs} - t_c z_{cs})PK_{t-1}K_{jcst-1}\, i_t \qquad c = c, n \quad j = 1\varepsilon I_{IND} \tag{A.3.41}$$

$$BQ_j = PKD_j KD_j - DC_j - IC_{cj} - R_{jc}^p \tag{A.3.42}$$

$$R_{jc}^f = t_c^f(1 - t_c^s)BQ_j \tag{A.3.43}$$

$$R_{jc}^s = t_c^s BQ_j \tag{A.3.44}$$

Noncorporate income tax base

$$BN_j = PKD_{jn}KD_{jn} - DN_j - IC_{nj} - R_{jn}^p \tag{A.3.44}$$

Individual capital income tax base

$$BE_j + BD_j = BQ_j - t_c BQ_j + BN_j + IC_{cj} + IC_{nj} \tag{A.3.45}$$

Individual capital income tax revenue from industry j

$$R_j^{If} = [\alpha^{DIV} t_c^{ef} + (1 - \alpha^{DIV})t_c^{gf}](BQ_j - R_{jc}^f) + t_n^{ef}\, BN_j + t_n^{ef}\,(IC_{cj} + IC_{nj}) \tag{A.3.46}$$

$$R_j^{Is} = [\alpha^{DIV} t_c^{es} + (1 - \alpha^{DIV})t_c^{gs}](BQ_j - R_{jc}^s) + t_n^{es}\, BN_j + t_n^{es}\,(IC_{cj} + IC_{nj}) \tag{A.3.47}$$

$$cash\_flow_j = BE_j + BD_j - R_j^{If} - R_j^{Is} + DC_j + DN_j \tag{A.3.48}$$

Household property taxes

$$R_h^p = t_h^p\, PK_{t-1}\,(K_{hst-1} + K_{hlt-1}) \tag{A.3.49}$$

$$debt_h = \beta_h\, PK_{t-1}\,(K_{hst-1} + K_{hlt-1}) \tag{A.3.50}$$

$$BH = 0 - R_j^p - debt_h\, i_t \tag{A.3.51}$$

* Revenue summed over all industries: Property; Capital income; Wealth taxes

$$RP = \sum_{j}^{NIND} R_{jc}^{p} + R_{jn}^{p} + R_{h}^{p} \tag{A.3.52}$$

$$RK^{f} = \sum_{j}^{NIND} R_{jc}^{f} + R_{j}^{If} + BH\, t_{h}^{ef} \tag{A.3.53}$$

$$+ tk^{f}(rBG + rBF)/(1 - tk) + tk^{f}(GINT^{rec} + YROW^{rec}) + (1 - tk^{f})PKD_{35}KD_{35}$$

$$RK^{s} = \sum_{j}^{NIND} R_{jc}^{s} + R_{j}^{Is} + BH\, t_{h}^{es} \tag{A.3.54}$$

$$+ tk^{s}(rBG + rBF)/(1 - tk) + tk^{s}(GINT^{rec} + YROW^{rec}) + (1 - tk^{s})PKD_{35}KD_{35}$$

$$RW^{f} = tw^{f}[\sum_{j}^{NIND} \sum_{s,l} (1 - ITC_{cs} - t_{c}z_{cs})PK_{t-1}K_{jcst-1} \tag{A.3.55}$$

$$+ (1 - ITC_{ns} - t_{n}z_{ns})PK_{t-1}K_{jnst-1} + BG_{t} + BF_{t}] \qquad f = f, s$$

$$Y^{I} = \sum_{j}^{NIND} IC_{cj} + IC_{nj} + debt_{h}\, i_{t} \tag{A.3.56}$$

A.4 The Government and pollution externalities

Tax rates.

$$tc_i = tc + tc^g \qquad i \varepsilon I_{COM} \tag{A.4.1}$$

$$tc_N = tc + tc^N \tag{A.4.2}$$

$$tc_K = tc + tc^K \tag{A.4.3}$$

$$tc_L = tc + tc^L \tag{A.4.4}$$

$$tx_i^v = \sum_{j=1} tx_j^{Xv} XP_{ij} \qquad i \varepsilon I_{IND} \quad j \varepsilon I_{EXT} \tag{A.4.5}$$

$$tx_i^u = \sum_{j=1} tx_j^{Xu} XP_{ij} \tag{A.4.6}$$

$$tx_i^{rv} = \sum_{j=1} tx_j^{Xv} XM_{ij} \tag{A.4.7}$$

$$tx_i^{ru} = \sum_{j=1} tx_j^{Xu} XM_{ij} \tag{A.4.8}$$

$$tt_i^{full} = tt_i + tx_i^v + \frac{tu_i + tx_i^u}{PO_i} \tag{A.4.9}$$

$$tl^0 = \sum_j PLD_j LD_j \, (1 - \frac{tl^a}{tl^m}) \tag{A.4.10}$$

$$tt_j = tt_j^f + tt_j^s \tag{A.4.11}$$

$$tl^m = tl^{mf} + tl^{ms} \tag{A.4.12}$$

$$tl^a = tl^{af} + tl^{as} \tag{A.4.13}$$

$$t_c^e = t_c^{ef} + t_c^{es} \qquad c = c, n, h \tag{A.4.14}$$

$$t_g^e = t_g^{ef} + t_g^{es} \qquad c = c, n, h \tag{A.4.15}$$

Stock-flow relations.

$$BG_t^f = BG_{t-1}^f + \Delta G^f + GFI + \Delta P_t^{BGF} + BG^{fdisc} \tag{A.4.16}$$

$$BG_t^s = BG_{t-1}^s + \Delta G^s \tag{A.4.17}$$

$$BG_t * = BG_{t-1} * - GFI - \Delta P_t^{BG*} \tag{A.4.18}$$

Federal Government revenue and expenditures on goods.

$$revenue^f = R\_SALES^f + R\_CON + R\_TARIFF + R\_K^f + R\_ITC + R\_L^f \quad \text{(A.4.18)}$$
$$+ R\_W^f + TAXN^f + TAXSS + R\_UNIT + R\_EXT + TLUMP$$

$$R\_SALES^f = \sum_j tt_j^f PO_j QI_j \quad \text{(a)}$$

$$R\_CON = \sum_{I_{COM}} tc_i PS_i C_i + tc_K \frac{KD_H}{KD} PII_t I_t^a \quad \text{(b)}$$

$$R\_TARIFF = \sum_i tr_i PM_i M_i \quad \text{(c)}$$

R_K^f is from (A.3.53)

$$R\_ITC = -\theta PII_t I_t^a \quad \text{(e)}$$

$$R\_L^f = tl^{af} wLS/(1 - tl^m) \quad \text{(f)}$$

$$R\_W^f = tw^f (PK.K + BG + BF)??????A.3.55 \quad \text{(g)}$$

$$R\_UNIT = \sum_j tu_j QI_j \quad \text{(h)}$$

$$R\_EXT = \sum_j tx_j^v (PI_j QI_j + PM_j M_j) + \sum_j tx_j^u (QI_j + M_j) \quad \text{(i)}$$

$$VGG^f = \Delta G^f + revenue^f$$
$$- r \frac{BG^f}{1 - tk} - GF\_INT^{rec} - GINTR - GF\_TRAN - GR - \theta PII_t I_t^a \quad \text{(A.4.18)}$$

State&Local Government revenue and expenditures on goods.

$$revenue^s = R\_SALES^s + R\_K^s + R\_L^s + R\_P \quad \text{(A.4.19)}$$
$$+ R\_W^s + TAXN^s$$

$$R\_SALES^s = \sum_j tt_j^s PO_j QI_j \quad \text{(a)}$$

R_K^s is from (A.3.54)

$$R\_L^s = tl^{as} wLS/(1 - tl^m) \quad \text{(b)}$$

$$R\_W^s = tw^s (PK.K + BG + BF)?????? \quad \text{(c)}$$

$$R\_P = t_c^p PII_t K_c + t_n^p PII_t K_n + t_h^p PII_t K_h \quad \text{(d)}$$

$$VGG^s = \Delta G^s + revenue^s$$
$$- r \frac{BG^s}{1 - tk} - GS\_INT^{rec} - GS\_TRAN \quad \text{(A.4.20)}$$

Total Government

$$GTRAN = GF\_TRAN + GS\_TRAN \tag{A.4.21}$$

$$GINT^{rec} = GF\_INT^{rec} + GS\_INT^{rec} \tag{A.4.22}$$

$$VGG = VGG^{f} + VGG^{s} \tag{A.4.23}$$

$$TAXN = TAXN^{f} + TAXN^{s} \tag{A.4.23}$$

$$VG_i = \alpha_i^G \, VGG \qquad i \varepsilon I_{INP} \tag{A.4.24}$$

$$G_i = VG_i / PS_i \tag{A.4.25}$$

$$VG \equiv (PS_1 G_1, \ldots, PS_{35} G_{35})'$$

$$G^P \equiv (G_1, \ldots, G_{35})'$$

$$G \equiv (G_1, \ldots, G_{35,} NCI_{G,} KD_G, LD_G)'$$

$$GFI = -GINTR - GR - GM \tag{A.4.26}$$

$$GINT^{rec} = GINT - \frac{r}{1 - tk} \, BG_{t-1} \tag{A.4.27}$$

Externalities

$$EXT_x = \sum_j XP_{jx} QI_j + \sum_i XM_{ix} M_i \qquad x \varepsilon I_{EXT} \tag{A.4.28}$$

A.5 The Rest-of-the-World

Non-competitive imports.

$$PNCI_j = e\,(1 + tr_j^n)PNCI_j^* \qquad j = 1, \ldots, 35, C, I, G \qquad \text{(A.5.1)}$$

Competitive imports and domestic output making up total supply:

$$QS_i = QS(\,QC_i, M_i) \qquad i \varepsilon I_{COM} \qquad \text{(A.5.2)}$$

with the price dual:

$$\ln PS_i = (\alpha_0^{Mi} + \frac{\beta_0^{Mi}}{1 + e^{-\mu(t-\tau)}}) \ln P^{Mi} + \tfrac{1}{2} \ln P^{Mi\prime} B^{Mi} \ln P^{Mi} \qquad \text{(A.5.3)}$$

$$\ln P^{Mi} \equiv (\ln PC_i, \ln PM_i)'$$

$$PM_i = e\,(1 + tr_i + tx_i^{rv})PM_i^* + tx_i^{ru} \qquad i \varepsilon I_{COM} \qquad \text{(A.5.4)}$$

$$SD^i \equiv \begin{bmatrix} PC_i QC_i / PS_i QS_i \\ PM_i M_i / PS_i QS_i \end{bmatrix} = \alpha_0^{Mi} + \frac{\beta_0^{Mi}}{1 + \exp(-\mu^{Mi}(t - \tau^{Mi}))} + B^{Mi} \ln P^{Mi} \qquad \text{(A.5.5)}$$

$$PS_i QS_i = PC_i QC_i + PM_i M_i \qquad i \varepsilon I_{COM} \qquad \text{(A.5.6)}$$

$$VQS \equiv (\,PS_1 QS_1, \ldots, PS_{35} QS_{35}\,)' \qquad \text{(A.5.7)}$$

$$VM \equiv (\,PM_1 M_1, \ldots, PM_{35} M_{35}\,)'$$

$$SM \equiv (\,SD_2^1, SD_2^2, \ldots, SD_2^{35}\,)'$$

$$M \equiv (\,M_1, M_2, \ldots, M_{35}\,)'$$

Exports.

$$X_i = X_i(Y^*, (1 + tr_i^*)PC_i / eP_i^*) \qquad i \varepsilon I_{COM} \qquad \text{(A.5.8)}$$

$$= EX_{i0}(Y^*) \left(\frac{(1 + tr_i^*)PC_i}{e} \right)^{\eta_i} + X_i^{tr} \qquad \text{(A.5.9)}$$

$$X_i^{tr} = \frac{PCC.XR}{\sum PC_i C_i} C_i \qquad \text{(A.5.10)}$$

$$EX_{it} = \alpha_i^X + \lambda_i^X \ln Y_t^* \qquad \text{(A.5.11)}$$

$$X \equiv (X_1, \ldots, X_{35})'$$

$$VX \equiv (PC_1 X_1, \ldots, PC_{35} X_{35})' \tag{A.5.12}$$

Current account and net foreign assets.

$$CA = \sum_i PC_i X_i + \frac{r}{1-tk} BF + YROW^{rec} - \sum_i PM_i M_i - \sum_j PNCI_j NCI_j \tag{A.5.13}$$
$$- \bar{i} BG^* - GR - CR$$

$$BF_t = BF_{t-1} + CA_t - GFI + BF^{disc} + \Delta P^{BF} \tag{A.5.14}$$

$$YROW^{rec} = YROW - \frac{r}{1-tk} BF \tag{A.5.15}$$

A.6 Markets

Final demands.

$$VFD_i = PS_i\,(C_i^P + I_i^P + G_i^P) + PC_i X_i \qquad i \varepsilon I_{COM} \tag{A.6.1}$$

$$VFD \equiv (VFD_1, \ldots, VFD_{35})'$$

$$= VC + VI + VG + VX$$

Supply equal demand for commodities.

$$PS_i QS_i = \sum_{j=1}^{35} PS_i QP_i^j + VFD_i \tag{A.6.2}$$

$$VQS = \mathbf{A}\,VQI + VFD \tag{A.6.3}$$

$$VQC = Diag(SM)\,VQS \qquad VQS = Diag(1/SM)\,VQC \tag{A.6.4}$$

$$Diag(1/SM)\,VQC - \mathbf{A}\,VQI = VFD$$

$$Diag(1/SM)\mathbf{M}'\,Diag(\iota + tt^{full})\,VQI - \mathbf{A}\,VQI = VFD$$

$$[Diag(1/SM)\mathbf{M}'\,Diag(\iota + tt^{full}) - \mathbf{A}\,]\,VQI = VFD \tag{A.6.5}$$

Saving-investment balance.

$$VII = S - (BG_t - BG_{t-1}) - (BF_t - BF_{t-1}) \tag{A.6.6}$$

Demand equal supply of capital.

$$PKD_j = \psi_j^K\,PKD \qquad j \varepsilon I_{BUY} \tag{A.6.7}$$

$$\sum_{j=1}^{C} PKD_j KD_j = PKD.\,KD \tag{A.6.8}$$

$$\sum_{j=1}^{C} \psi_j^K KD_j = KD = \psi^K K_{t-1} \tag{A.6.9}$$

Demand equal supply of labor.

$$PLD_j = \psi_j^L\,\frac{w}{(1 - tl^m)} \qquad j\ \varepsilon I_{BUY} \tag{A.6.10}$$

$$w^{LE} = \psi_C^L\,w \tag{A.6.11}$$

$$(1 - tl^m) \sum PLD_j LD_j = w\,LS = w\,(LH - \psi_C^L LEIS) \tag{A.6.12}$$

$$\sum_{j=1}^{G} \psi_j^L LD_j = LS \qquad \text{(A.6.13)}$$

A.7 Steady state equilibrium

$$Prices_T = Prices_{T-1} \quad \text{(A.7.1)}$$

$$Quantities_T = Quantities_{T-1} \quad \text{(A.7.2)}$$

$$\Delta G_T = 0 \quad \text{(A.7.3)}$$

$$CA_T = 0 \quad \text{(A.7.4)}$$

$$r_t = \rho \quad \text{(A.7.5)}$$

$$\psi^I I_T^a = \delta K^T \quad \text{(A.7.6)}$$

A.8 GLOSSARY

A.8.1 Values and other variables:

A		IO Use matrix; the use of commodities by each industry
A_j	$j \varepsilon I_{IND}$	Columns of **A**
A_{ij}	$i \varepsilon I_{COM} \quad j \varepsilon I_{IND}$	Share of input i in producing output j
BF		Net US private sector claims on rest-of-world
BF^{disc}		Stock-flow discrepancy in the US external accounts
BG		Government debt to domestic households
BG^f	$f = f, s$	Ditto; federal, state&local
BG^{disc}		Stock-flow discrepancy in the US government accounts
$BG*$		Government debt to rest-of-world
CA		Current account surplus of the US
CR		Households transfer to rest-of-world
$cash\_flow$	$j \varepsilon I_{IND}$	Capital income after taxes from business
EX_{it}		Exogenous projected exports.
GFI		Government net foreign investment
$GINT$		Government interest payments on public debt to households (including social insurance funds.)
Gf_INT	$f = F, S$	Ditto; federal, state&local
$GINT^{rec}$		Arbitrage adjustment for interest income on government bonds
Gf_INT^{rec}	$f = F, S$	Ditto; federal, state&local
$GINTR$		Government interest payments to rest-of-world
GM		Government net imports
GR		Government transfers to rest-of-world
$GTRAN$		Government transfers to households
M_k		Expenditures by household k
MF		Full expenditures (including leisure)
R_j^I		Individual capital income tax revunue from industry j
SD^i	$i \varepsilon I_{COM}$	Shares of domestic output, imports in total supply of i
SF		Vector of shares of commodities and leisure in full consumption
SI^m	$m \varepsilon I_{INV}$	Shares of investment at node m
SM		Vector of shares of imports in total supply.
SN^m	$m \varepsilon I_{CNODE}$	Shares of consumption at node m
SP^{jm}	$j \varepsilon I_{IND} \quad m \varepsilon I_{PNODE}$	Shares of production at node m of industry j
S		Savings
$TLUMP$		Lump sum tax
$TAXN$		Non-tax receipts of the government
$TAXN^f$	$f = f, s$	Ditto; federal, state&local
$TAXSS^f$	$f = f, s$	Govt investment income of soc ins funds
VC		Vector of values of household purchases of commodities
VFD		Vector of values of final demand for commodities
VG		Vector of values of government demand for commodities

VGG		Value of government purchases
VII		Value of domestic private investment
VI		Vector of values of investment inputs
VN		Vector of values of household purchases of NIPA commodities
VP^{j}	$j \varepsilon I_{IND}$	Vector of values of inputs into industry j
VQC		Vector of values of domestic commodity output
VQI		Vector of values (to producer) of domestic industry output
VQI^{tt}		Vector of values of domestic industry output inclusive of sales tax
VQS		Vector of values of total commodity supply
VX		Vector of values of commodity exports
W		Tangible wealth of private sector (households)
WF		Full wealth of private sector (households)
XR		Travel exports: Expenditures by foreign tourists in U.S.
Y		Income
YF		Full income (including imputations on leisure)
$YROW$		Net income from rest-of-world
$YROW^{rec}$		Arbitrage adjustment for income from rest-of-world
$Y*$		Exogenous projected rest-of-world income
Y^{I}		Interest from debt portion of claims on all capital
ΔG		Government deficit
ΔG^{f}	$f = f, s$	Government deficit; federal, state&local
ΔP^{BF}		Capital gains on net foreign assets
ΔP^{BG}		Capital gains on government bonds
ΔP^{BG*}		Capital gains on government liabilities to ROW
ψ^{I}		Aggregation constant of investment goods
ψ^{PK}		Aggregation constant of price of capital

A.8.2 Quantities:

CC		Aggregate consumption (commodities)
C^{P}		Vector of quantities of consumption of produced commodities.
C		Vector of consumption, commodities & non-produced goods.
C_i	$i \varepsilon I_{INP}$	Consumption of IO commodity i
EXT_x	$x \varepsilon I_{EXT}$	Quantity of Externality of type x
F		Full consumption (commodities and leisure)
G^{P}		Vector of government purchases of commodities.
G		Vector of government purchases, commodities and non-produced goods.
G_i	$i \varepsilon I_{NCOM}$	Government purchases of commodity i
I^{a}		Aggregate investment in domestic capital stock
I		Vector of commodities used in aggregate investment.
I^{m}	$m \varepsilon I_{INV}$	Investment aggregate m

I_i^f	$i \varepsilon I_{NCOM}$	Investment of commodity i in fixed investment
I_i	$i \varepsilon I_{NCOM}$	Investment of commodity i in domestic capital stock
K		Capital stock located in the U.S.
KD		Quantity of total capital input normalized such that its rental price is one
KD_j	$j \varepsilon I_{NBUY}$	Quantity of capital input into sector j
LD_j	$j \varepsilon I_{NBUY}$	Quantity of labor input into sector j
$LEIS$		Leisure time
LH		Time endowment of economy
LS		Labor supply
M		Vector of competitive imports.
M_i	$i \varepsilon I_{COM}$	Imports of (competitive) commodities
N^{eq}		Number of household equivalent members in economy
N^m	$m \varepsilon I_{CNODE}$	Consumption of NIPA aggregate m
N_i	$i \varepsilon I_{PCE}$	Consumption of NIPA commodities
NCI_j	$j \varepsilon I_{NBUY}$	Non-competitive imports into sector j
QC_i	$i \varepsilon I_{COM}$	Total domestic output of commodity i
QI_j	$j \varepsilon I_{IND}$	Output of industry j
QP^{jm}	$j \varepsilon I_{IND}$ $m \varepsilon I_{PNODE}$	Aggregate input m into industry j
QP_i^j	$i \varepsilon I_{COM}$ $j \varepsilon I_{IND}$	Input of commodity i into industry j
QS_i	$i \varepsilon I_{COM}$	Total supply of commodity i
X		Vector of exports.
X_i	$i \varepsilon I_{COM}$	Exports of commodity i
X_i^{tr}	$i \varepsilon I_{COM}$	Travel exports of commodity i

A.8.3 Prices:

e		"exchange rate"
i^*		interest rate on private US owned foreign assets
r		After tax interest rate used in Euler eqn.
r_{jc}	$j \varepsilon I_{IND}$	average (over equity, debt) rate of return to corporate capital
r_{jn}	$j \varepsilon I_{IND}$	average rate of return to noncorporate capital
r_h		average rate of return to household capital
i		Interest rate on debt
w		Price of total hours (work and leisure)
$\bar{w}$		After tax average wage rate
w^{LE}		Price of leisure
P_i^*	$i \varepsilon I_{COM}$	World price for US exports.
P^{Hm}	$m \varepsilon I_{CNODE}$	Vector of prices at node m of consumption function
P^{im}	$m \varepsilon I_{INV}$	Vector of prices at node m of investment function
P^{Pjm}	$j \varepsilon I_{IND}$ $m \varepsilon I_{PNODE}$	Vector of prices at node m of industry j's production function
PC_i	$i \varepsilon I_{COM}$	Price of domestically produced commodities
PF		Price of full consumption
PI_j	$j \varepsilon I_{IND}$	Price of industry output paid by buyers

PII		Price of aggregate investment goods
PII^m	$m \varepsilon I_{INV}$	Price of investment aggregate m.
PII_{mi}	$mi \varepsilon I_{INVm}$	Union of above aggregate investment prices and supply prices.
PK		Price of capital stock
PKD_j	$j \varepsilon I_{BUY}$	Rental price of capital paid by producer
PLD_j	$j \varepsilon I_{BUY}$	Price of labor paid by employers
PM_i	$i \varepsilon I_{COM}$	Price of non-competitive imports paid by importers
PN_n	$n \varepsilon I_{NIPA}$	Price of NIPA PCE commodity
PN^m	$m \varepsilon I_{CNODE}$	Price of consumption aggregate m
PN_{mi}	$mi \varepsilon I_{CNODEm}$	Union of above 2 sets of consumption prices
$PNCI_i$	$j \varepsilon I_{BUY}$	Price of imports paid by importers
PO_j		Price of industry output received by producer
PP^{jm}	$j \varepsilon I_{IND}$ $m \varepsilon I_{PNODE}$	Price of aggregate input m into industry j
PP^j_{mi}	$mi \varepsilon I_{PNODEm}$	Union of above set of aggregate production prices and prices of inputs.
PS		Vector of supply prices.
PS_i	$i \varepsilon I_{COM}$	Price of commodities to buyers
PS_i^C	$i \varepsilon I_{COM}$	Prices of commodities paid by the household sector

A.8.4 Parameters of behaviorial equations:

ρ		Pure rate of time preference
σ		Household intertemporal elasticity of substitution
α^F		Shares (at unit prices) of commodities and leisure in full consumption
B^F		Share elasticity of components of full consumption (w.r.t. prices)
α_0^F		Shares (at $t = -\infty$) of commodities and leisure in F
β_0^F		Trend coefficient of commodities and leisure in F
μ^F		Slope of logistic curve in F
τ^F		Mid-point of logistic curve
α^{Hm}	$m \varepsilon I_{CNODE}$	Shares (at unit prices) of consumption at node m
B^{Hm}		Share elasticity of consumption (w.r.t. prices) at node m
B_{pA}		Coefficients on demographic characteristics of CC function
ξ^d		Distribution coefficient of CC function
ξ^L		Vector of demographic dummies
α_0^j	$j \varepsilon I_{IND}$	Cost function constant
α^{Pjm}	$j \varepsilon I_{IND}$ $m \varepsilon I_{PNODE}$	Shares (at unit prices) of of inputs into industry j at node m
B^{Pjm}		Share elasticity of input demands (w.r.t. prices) at node m
B^j_{pt}	$j \varepsilon I_{IND}$	Biases of technical change
μ^j		Slope of logistic curve representing index of technology

τ^{j}		Mid-point of logistic curve
A^{j}_{KD0}	$j \varepsilon I_{IND}$	Constant of industry capital input price function
A^{j}_{KD}	$j \varepsilon I_{IND}$	Shares (at unit prices) of inputs of industry capital input
B^{j}_{KD}	$j \varepsilon I_{IND}$	Share elasticity of components of industry capital input
α^{jc}_{KD}	$j \varepsilon I_{IND}$	Shares (at p=1) of components of indus corporate cap input
B^{jc}_{KD}	$j \varepsilon I_{IND}$	Share elasticity of components of indus corporate cap input
α^{jn}_{KD}	$j \varepsilon I_{IND}$	Shares (at p=1) of components of indus noncorp cap input
B^{jn}_{KD}	$j \varepsilon I_{IND}$	Share elasticity of components of indus noncorporate cap input
α^{h}_{KD}		Shares (at p=1) of components of household capital input
B^{h}_{KD}		Share elasticity of components of household capital input
α^{IY}		Share of inventory investment in total investment
α^{IY}_{i}	$i \varepsilon I_{COM}$	Share of inventory investment going to commodity i
α^{Im}	$m \varepsilon I_{INV}$	Shares (at unit prices) of commodities at investment node m
B^{Im}	$m \varepsilon I_{INV}$	Shares elasticity of components of totalinvestment at node m
α^{I}		Constant term in logistic curve of aggregation constant converting investment into capital, ψ^{I}
β^{I}		Trend coefficient of logistic curve, ψ^{I}
μ^{I}		Slope of logistic curve, ψ^{I}
τ^{I}		Mid-point of logistic curve, ψ^{I}
α^{PK}		Constant term in logistic curve of aggregation constant converting price of investment into price of capital, ψ^{PK}
β^{PK}		Trend coefficient of logistic curve, ψ^{PK}
μ^{PK}		Slope of logistic curve, ψ^{PK}
τ^{PK}		Mid-point of logistic curve, ψ^{PK}
α^{Mi}	$i \varepsilon I_{COM}$	Shares (at unit prices) of domestic commodities and imports in total supply
α^{Mi}_{0}		Shares (at t=$-\infty$) of domestic commodities and imports
β^{Mi}_{0}		Trend coefficient of domestic commodities and imports
μ^{Mi}		Slope of logistic curve representing import penetration
τ^{Mi}		Mid-point of logistic curve
B^{Mi}	$i \varepsilon I_{COM}$	Shares elasticity of components of total supply
η^{i}	$i \varepsilon I_{COM}$	Export price elasticities
α^{G}_{i}	$i \varepsilon I_{INP}$	Share of government expenditures on i
δ_{cs}	c=c,n,h	Rate of depreciation of short-lived capital stock
δ_{ch}	c=c,n,h	Rate of depreciation of long-lived capital stock
β_{jc}		corporate debt-equity ratio, industry j

β_{jn}		noncorporate debt-equity ratio, industry j
β_h		debt-equity ratio, household
α^{DIV}		dividend-payout ratio
ψ^K		Aggregation constant of capital services
ψ_j^K	$j \varepsilon I_{BUY}$	Aggregation constants of capital
ψ_j^L	$j \varepsilon I_{BUY}$	Aggregation constants of labor
ψ_C^L		Aggregation constants of leisure
λ_i	$i \varepsilon I_{IND}$	Shocks to production function
λ^I		Shocks to investment function
XP_{ix}	$i \varepsilon I_{IND} \quad x \varepsilon I_{EXT}$	Production externalities
XM_{ix}	$i \varepsilon I_{COM} \quad x \varepsilon I_{EXT}$	Import externalities
H		Bridge matrix linking NIPA "Personal Cons. Expenditures" commodities to IO commodities
M		IO Make matrix; the contribution of each industry to each commodity

A.8.5 Tax rates:

t_c		Tax rate on corporate capital income (federal + S&L)
t_c^f	$f = f, s$	Statutory tax rate on corp. capital income; federal, S&L
ITC_{cs}	$c = c, n \; s = s, l$	Investment tax credit (corp,noncorp; short,long)
z_{cs}	$c = c, n \; s = s, l$	Depreciation allowances for $1 of investment
t_c^p		Property tax rate; corporate
t_n^p		Property tax rate; noncorporate
t_h^p		Property tax rate; household capital
t_c^g	$c = c, n, h$	capital gains tax (corp, noncorp, household)
t_c^e	$c = c, n, h$	tax on equity income (corp, noncorp, household)
t_c^{earn}		average tax on personal corporate capital income
γ_c^p	c=c,n,h	deduction of property taxes (=1 in version 9)
γ_c^i	c=c,n,h	proportion of interest payments deducted before tax
γ_c^d		proportion of dividends deducted before tax on corp.
γ_c^g	c=c,n	proportion of capital gains on corp equities excluded from individual income for tax purposes.
dhi		proportion of inflation premium in interest determined by indexing rule of household interest expense
t_h		tax rate on household income used to adjust deductions
tc_i	$i \varepsilon I_{COM}$	Total tax rate on consumption commodity
tc		Consumption tax rate
tc^g		Consumption tax on goods only

tc^N		Consumption tax on imports only (NCI)
tc^K		Consumption tax on household capital input
tc^L		Consumption tax on private household labor.
tl^a		Average tax rate on labor income
tl^{af}	$f = f, s$	Average tax rate on labor income; federal, state
tl^m		Marginal tax rate on labor income
tl^{mf}	$f = f, s$	Marginal tax rate on labor income; federal, state
tl^0		Implied tax rate on labor income at zero income
tr_i	$i \varepsilon I_{COM}$	Tariff rate on competitive imports
tr_i^n	$i \varepsilon I_{BUY}$	Tariff rate on non-comp imports
tr_i *	$i \varepsilon I_{COM}$	World tariff rate on US exports
tt_j	$j \varepsilon I_{IND}$	Indirect business tax (sales tax)
tt_j^f	$f = f, s$	Indirect business tax; federal, state&local
tt_j^{full}	$j \varepsilon I_{IND}$	The full tax rate on sales.
tu_i	$I \varepsilon I_{IND}$	Unit tax on quantities sold
tx_i^u	$i \varepsilon I_{IND}$	Total Unit externalities tax on quantities sold
tx_i^v	$i \varepsilon I_{IND}$	Total Externalities tax on sales
tx_i^{ru}	$i \varepsilon I_{COM}$	Total Unit externalities tax on quantities imported
tx_i^{rv}	$i \varepsilon I_{COM}$	Total Externalities tax on imports
tx_j^{Xu}	$j \varepsilon I_{EXT}$	Tax on one unit of externality j
tx_j^{Xv}	$j \varepsilon I_{EXT}$	Tax on one dollar of externality j
θ		Rate of investment tax credit (post sample)
tw^f	$f = f, s$	Wealth tax rate (estate taxes)